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Switching Supervisory Control: Adaptive and Input-constrained Systems

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Abstract

Switching Supervisory Control: Adaptive and Input-constrained Systems

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This thesis deals with the application of logic-based switching in adaptive and predictive control schemes. Each control system under study consists of a switching supervisory control architecture whereby a controller, selected from a finite family of pre-designed candidate controllers, is at any time switched-on in feedback to the plant based on available measurements.

In Part I of this thesis, attention is devoted to the use of logic-based switching in adaptive control of plants with large modelling uncertainties which render robust control design tools inadequate. In this respect, we study the problem of how to on-line infer stability of a potential feedback-loop made up by a given candidate controller interconnected in feedback with an uncertain plant, while the latter is possibly driven by a different controller. Assuming that a nominal model is associated to the candidate controller in such a way that the resulting so-called *tuned-loop* be stable, an answer is provided by sufficient conditions expressed in terms of percentage measures of discrepancy between the potential loop and the tuned one associated to the given candidate controller. It is shown that the proposed solution, while retaining the high switching performance capability inherent to model-based control, also enjoys model-free guaranteed stability properties, as stability is proved to hold under the only assumption that a stabilizing candidate controller exist. Attention is also devoted to the analysis of situations where the supervisory unit operates in a noisy environment.

Part II of this thesis deals with the set-point tracking problem of critically unstable linear systems subject to positional and/or incremental input constraints and also affected by persistent disturbances of unknown arbitrary magnitude. The proposed solution is realized via a supervisory switching control scheme whereby the feedback-gain selection is made in accordance with a predictive control philosophy, and each candidate feedback-gain is tuned on to a different control-horizon in a receding-horizon control sense. It is shown that the proposed solution ensures global asymptotic and offset-free tracking in the presence of constant disturbances and set-point, as well as finite l_∞ induced gain of the disturbance-to-state map in the presence of time-varying disturbances and set-point, while preserving the fulfillment of control-increment saturation constraints. It is also shown that the predictive switching logic scheme considered, provides, relatively to alternative approaches yielding comparable feasibility/performance properties, a computationally affordable solution.

Per Giuditta Scarselli.

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Contents

Chapter 1

Introduction

In the name of God, stop a moment, cease your work, look around you.

Lev Tolstoj (1828-1910)

In recent years, the idea of employing logic-based switching in control has attracted significant research efforts. In several control engineering applications, switching between controllers is a subject of special interest: the process to be controlled might change modes of operation so as to render no single controller capable by itself of guaranteeing closed-loop stability [Mos95, IS96, HL01a, AD08]. Switching between controllers might be crucial in protecting the process against wind-up effects [GOMW00, GGS01]. Changes in the dynamics of the process under control and/or in the character of the disturbances might require prompt changes in the control action in order to maintain satisfactory closed-loop performance [NB94, HL01a, HM02].

Typically, a switching control scheme consists of a *modular* architecture that separates an inner loop, the ordinary control system, from an outer loop, which comprises a "high-level" unit, whose task is to orchestrate the switching. Historically, one of the first schemes employing logic-based switching in control is – as we know it today – the so-called Gain-Scheduling control [SA90, RS00]. In Gain-Scheduling, as depicted in Fig. 1.1, the outer loop consists of a supervisory unit, which accomodates changes in the control action by means of a look-up table associating the candidate controller parameters θ_i's to available auxiliary measurements z. Architectures such as the one of Fig. 1.1 provide greater flexibility in applications since the controller parameters can be computed off-line; further, between switching instants, the dynamic of the supervisor does not affect the inner loop, and the overall system analysis only relies on the properties of its individual parts.

Because of the above motivations, during the last decades, many research efforts have been devoted to the development of switching control schemes akin to, or based upon, Gain-Scheduling strategies: Adaptive switching control (ASC) – the monograph [Lib03] and the survey papers [KS01, HL01a, AD08] provide an overview of the topic – has recently emerged as

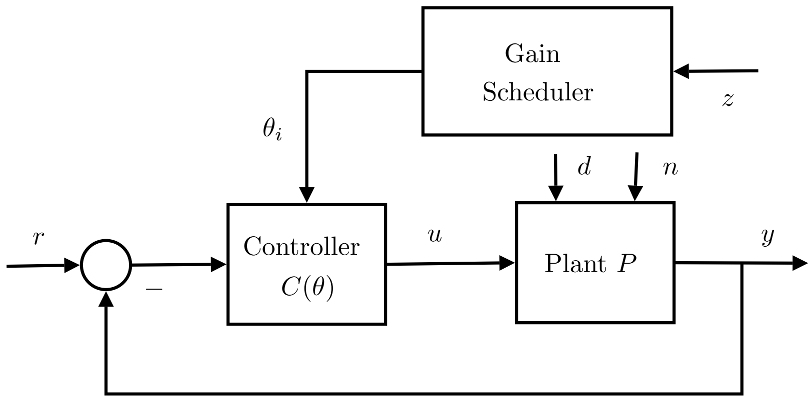


Figure 1.1: Typical Gain-Scheduling control scheme.

an alternative to conventional continuous adaptation [Mos95, IS96]. ASC resembles an adaptive variant of classic Gain-Scheduling control in that it aims at extending the latter to applications where the supervisory unit has access to I/O records of the uncertain plant [1]; Gain-Scheduling control for Networked Control Systems is now attracting a great deal of attention [TC04a, TC04b, NPTL05, LC06]; Gain-Scheduling predictive control is another relevant Gain-Scheduling variant, which has found application in control of systems subject to input/state constraints [ARS96, SHS00a, CFF01, CFZ03, Mos05].

In this thesis, variants of Gain-Scheduling are applied to adaptive and predictive switching supervisory control schemes. Specifically, Part I of this thesis concerns adaptive switching control of (possibly nonlinear) uncertain plants, while, in Part II of this thesis, predictive switching control of input-constrained critically unstable linear systems [2] is considered. In both the mentioned problems, particular attention will be devoted to study logic-based switching algorithms which can operate in noisy environments, a situation the latter in which the presence of noises/disturbances acting on the plant and corrupting the information available to the supervisory unit, adds substantial complexity in achieving stability guarantees. The remainder of this chapter briefly formulates the overall problems of interest and sums up the original contribution of this thesis. A more detailed analysis of each topic into the perspective of previous reasearch can be found in the body of the thesis.

[1] In fact, Gain-Scheduling is, at times, considered as a type of "open-loop" adaptive control [RS00], since there is no feedback which can compensate for an incorrect schedule.

[2] The so-called ANCBI (Asymptotically Null-Controllable with Bounded Input) systems.

1.1 Model-based Adaptive Switching Control

One of the approaches for controlling uncertain plants relies on the introduction of adaptation in the feedback loop [Mos95, IS96]. In particular, control of plants with large dynamic uncertainties requires adaptation in the feedback loop whenever robust control turns out to be inadequate. In recent years, *Adaptive Switching Control* (ASC) has emerged as an alternative to conventional continuous adaptation. As noted, ASC resembles an adaptive variant of classic gain-scheduling control in that it aims at extending the latter to applications where the "high-level" unit, called the supervisor, has access to I/O records of the uncertain plant. In ASC, the supervisor switches-on at any time in feedback with the uncertain plant P one controller C from a family $\mathscr{C}$ of candidate controllers using recorded plant I/O data. These data are processed in such a way to enable the supervisor of deciding whether the currently switched-on controller is adequate, and, in the negative, replace it by another candidate controller.

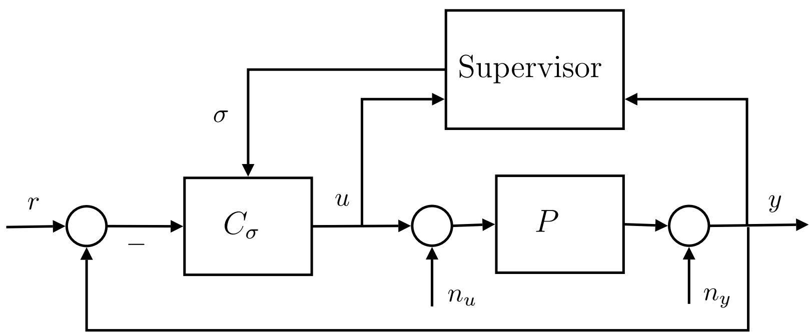


Figure 1.2: Typical ASC arrangement.

Let $\mathbb{S}$ be the linear space of all the real-valued sequences on $\mathbb{Z}_+ := \{0, 1, \ldots\}$. In the simplest nontrivial case considered throughout Part I of this thesis, the uncertain plant P consists of a strictly causal SISO discrete-time time-invariant dynamic system $P : \mathbb{S} \to \mathbb{S}$. Along with the uncertain plant P, a finite family $\mathscr{C} = \{C_i, i \in \underline{N}\}$, $\underline{N} := \{1, \ldots, N\}$, of one-degree-of-freedom LTI controllers C_i is available [3]. Each candidate C_i is described by a difference equation of the form

$$C_i : r_i(d)u(t) = S_i(d)(r(t) - y(t)), \qquad t \in \mathbb{Z}_+ \tag{1.1}$$

with $r_i(d)$ and $S_i(d)$ coprime polynomials in the unit backward shift operator d. The *scheduling* task (*i.e.* when to substitute the acting controller) and the *routing* task (*i.e.* which controller to switch on) are carried out in real-time by monitoring N test functionals V_i, $i \in \underline{N}$, each related to a different candidate controller. Accordingly, denoting by $\sigma(t)$, $\sigma(t) \in \underline{N}$, the index that identifies the

[3] In [HLM+01, SS08], the class of candidate controllers is allowed to have arbitrary cardinality.

controller connected in feedback to P at time t, the overall problem of interest consists of suitably selecting a test functional $V := \{V_i,\ i \in \underline{N}\}$ and a switching logic

$$\sigma(t) = \ell(V(t), \sigma(t-1)) \tag{1.2}$$

in such a way that the switched system, denoted by $(P/C_{\sigma(\cdot)})$, have desired stability/performance properties. By assuming that the input and the output of the plant are affected by unknown additive disturbances n_u and n_y, the ASC system of interest, as depicted in Fig. 1.2, is as follows [4].

$$\begin{cases} y(t) = P[u + n_u](t) + n_y(t) \\ u(t) = C_{\sigma(t)}[r - y](t) \end{cases} \quad , \qquad t \in \mathbb{Z}_+ \tag{1.3}$$

The main distinguishing feature of ASC over conventional adaptation is that controller selection relies on logic-based switching rather than continuous adaptation, which usually employs recursive parameter tuning schemes [Mos95, IS96]. It is well known that ASC, while retaining the fundamental ideas of adaptive control, enjoyes several potential advantages over traditional forms of adaptive control [NB94, Mor95, NB97, HLM03b]: ASC allows fast adaptation, since the controller selection is performed in a discontinuos fashion. ASC modularity allows for the integration into the inner loop of pre-existing control structures (or controllers which can be off-line synthesized regardless of their computational complexity), with no need of having a continuously parameterized family of controllers. Further, such a feature allows one to circumvent shortcomings in the controller synthesis, such as danger of stabilizability loss of the identified model, which might be encountered in formulating adaptive control as a recursive tuning control problem. Finally, in ASC, between switching instants, the dynamic of the supervisor does not affect the inner loop behavior, an advantageous feature from the analysis viewpoint.

Despite the vast literature on the subject $-$ *e.g.* see the monograph [Lib03] and the survey papers [KS01, HL01a, AD08] $-$ no sigle approach has so far distinctly encountered unanimous consensus among the control engineering community. Part I of this thesis addresses one of the key issues in ASC: how to synthesize test functionals for monitoring stability of the switched system and inferring the suitably of the candidate controllers relatively to P. In this respect, current approaches to ASC are traditionally grouped into the following classes: *Performance-based* control [Nus83, Mar85, FB86, ST97, SWPS07] is a model-free approach, whereby the supervisory unit orchestrates the switching by means of test functionals purely based on performance variables, *viz.* without trying to estimate, in accordance with the plant I/O data, the process model parameters. In contrast with it, *Model-based* control [NB94, Mor95, NB97, ZMF00, PJ01, HLM$^+$01, FAP06, KI08] performs the switching via test functionals based on identification errors, *viz.* the switching also

[4] Hereafter, $v = \mathfrak{P}[u]$ denotes a causal mapping from the input vector u to the output vector v.

relies on a nominal model distribution approximating the plant uncertainty set [5]. The main positive feature of the schemes in the first group is the fact that they can ensure stability of the switched system under very mild conditions. This is in particular the case for the *Unfalsified ASC* (UASC) of [ST95, ST97, SWPS07, SS08], whereby stability and finite convergence for switching are guaranteed under the minimal conceivable requirement, *viz.* the existence of a stabilizing candidate controller. On the other hand, current UASC schemes are at times criticized as they need not provide protection against the temporary insertion in the loop of destabilizing controllers [WE08, AD08, BBMT09]. Hence, in UASC, significant initial transients and temporary trends to divergence are typically experienced before the final controller is switched on. This situation looks antithetic to Model-based control, henceforth *Multi-model ASC* (MMASC), where transients can be made small at the cost of dense nominal model distributions. However, if the latter condition is not enforced, neither convergence to a final controller nor boundedness are in general ensured [HLM$^+$01, SWPS07, AD08, BBMT09].

The main contribution of Part I of this thesis is to develop novel model-based UASC schemes whereby the positive features of the two mentioned approaches might coexist and their respective deficiencies moderated. It will be shown that the proposed scheme, called *multi-model UASC*, can moderate, as in MMASC, the chance that destabilizing controllers be switched-on and, hence, reduce both the magnitude and time durations of "learning" transients after start-up, while, in contrast with pre-existing multi-model based methods, stability in-the-large and finite convergence for switching are guaranteed, as in UASC, under the only assumption that a stabilizing candidate controller exist. In other terms, the proposed solution, while retaining the high switching performance capability of MMASC, also enjoys UASC model-free guaranteed stability properties.

Analysis of multi-model UASC is carried out throughout Chapter 3 and Chapter 4. Conversely, Chapter 2 introduces the necessary background and put it into perspective with subsequent analysis. In this respect, it is worth saying that, although the main result of Part I of this thesis is in the mentioned multi-model approach to UASC, the developments of Chapter 2, while providing a quick review of UASC, also present further extensions of the results of [SWPS07].

1.1.1 Analysis of unfalsified ASC in a noisy environment

In UASC, the feedback adaptation task of classic adaptive control is replaced by controller falsification. The active controller can be falsified via a comparative experiment, by resorting to the *virtual reference* concept introduced in [ST95, ST97]. Specifically, at every time-step, and for each

[5] In Model-based control, the controller selection typically consists of switching on, at every time-step, the candidate controller whose related nominal model best approximates the uncertain plant. This is, in practice, an extension from tuning to switching of the *Certainty Equivalence Principle* [Mor95, NB97, Hes98].

candidate controller C_i, $i \in \underline{N}$, one solves in real-time the difference equation

$$r_i(d)u(t) = S_i(d)(v_i(t) - y(t)) \tag{1.4}$$

with respect to the virtual reference v_i, which can be done provided that C_i is causal, and stably causally invertible (CSCI), that is, $S_i(d)$ strictly Hurwitz. In words, v_i equals the fictitious reference that, if injected into the feedback system (P/C_i), would reproduce $z := \{u, y\}$, *viz.*

$$z(t) := (P/C_{\sigma(\cdot)})[w](t) = (P/C_i)[w_i](t), \qquad t \in \mathbb{Z}_+ \tag{1.5}$$

where $w = \{r, n_u, n_y\}$ and $w_i := \{v_i, n_u, n_y\}$. The first use of the virtual reference in ASC dates back to [ST95, ST97], where the test functional V_i was set so as to provide an estimate from below of the induced gain of the $l_2 - l_2$ map embodied by (P/C_i) from v_i to $[\, u \ y - v_i \,]'$,

$$V_i(t) = \mathfrak{V}_i(z^t, C_i) := \max\left\{ \frac{\left\|(\Sigma_{*/i}[v_i])^\tau\right\|_2}{\|v_i^\tau\|_2}, \ \tau = 0, \cdots, t \right\}, \qquad i \in \underline{N} \tag{1.6}$$

with $s^t := \{s(0), s(1), \ldots, s(t)\}$, $s \in \mathbb{S}$, and $\Sigma_{*/i} : v_i \to [\, u \ v_i - y \,]'$ denoting the mixed sensitivity of (P/C_i). Based on the innovative approach of [ST95, ST97], several UASC schemes have been thereafter proposed [JS99, CF99, BTS01, MA01], along with stability analysis [SWPS07, SS08]. However, to date, issues of applicability of UASC in noisy environments have been always overlooked. In this respect, Theorem 2.2.1 and Theorem 2.4.1 in Chapter 2 provide a generalization of the analysis of [SWPS07]. In particular, Theorem 2.2.1 provides conditions under which, in the presence of disturbances, it is possible to ensure switched system stability as well as finite convergence for switching for every plant initial condition, reference $r \in \mathbb{S}$, and any signal norm $\|\cdot\|$ such that

(a) if the sequence s converges exponentially to zero then $\|s^t\|$ is bounded, *i.e.*, there exists a nonnegative finite real α such that $\|s^t\| \leq \alpha$ for any $t \in \mathbb{Z}_+$.

Theorem 2.4.1 further generalizes Theorem 2.2.1, by providing conditions under which, for every plant initial condition and reference $r \in \mathbb{S}$, the switched system turns out to be jointly input-output stable with respect to different norms. The rationale behind the introduction of generalized norms and mixed-norm test functionals stems from the fact that in UASC, as discussed in Section 2.3, stability results are restricted to the switched system $(P/C_{\sigma(\cdot)})$, and no claim on stability of the unswitched final closed-loop can be advanced. This is an issue which has often been disregarded in the literature [DAL07, AD08], and deserving careful and detailed attention.

1.1.2 Multi-model unfalsified adaptive switching control

The main positive feature inherent to all early UASC schemes [ST95, ST97, SWPS07] is the fact that boundedness of the map from the reference r to the plant I/O data z can be ensured under the minimal conceivable requirement, *viz.* the existence of a stabilizing candidate controller. Such a feature, along with the fact that the plant need not to be linear, makes, from the robustness viewpoint, UASC a significant improvement over pre-existing MMASC schemes, whose stability is typically only guaranteed if the unknown plant is tightly approximated by at least one candidate model [Mor95, Hes98, HLM$^+$01, AD08]. However, there are futher aspects in an ASC system which deserve consideration. Particularly, an important *quality factor* of a test functional is its capability of inferring if a candidate controller would make the loop unstable once switched-on in feedback to the plant. As exemplified in Section 3.1 and also pointed out in [DAL07, WE08, BBMT09], such a feature cannot be ensured by test functionals of the type (1.6) whereby no plant model is used.

The main contribution of Chapter 3 is to show that it is possible to construct UASC schemes based on multiple-models, hence the name Multi-model UASC, in such a way that the resulting supervisory switching control scheme not only can moderate, as in MMASC, the chance that destabilizing controllers be switched-on, but, being intrinsically based on a stability test, also retains the desirable model-free asymptotic properties of UASC. Let $\mathscr{M} := \{M_i, i \in \underline{N}\}$ be a finite family of N strictly causal LTI dynamic models M_i approximating the plant uncertainty set,

$$M_i(d) := B_i(d)/a_i(d), \qquad i \in \underline{N} \tag{1.7}$$

where $B_i(d)$ and $a_i(d)$ are polynomials with strictly Hurwitz g.c.d., $a_i(0) = 1$. The M_i's, along with the associated C_i's, form a finite family of (internally stable) "*tuned-loops*" or "*reference-loops*"

$$\mathscr{R} := \{(M_i/C_i), \ \ i \in \underline{N}\}$$

each designed to fulfill desirable prescriptions. One of the main steps in multi-model UASC is to carry out a *reference-loop identification* task (Fig. 1.3). This task consists of selecting a candidate controller C_σ in such a way that (P/C_σ) behave as closest as possible to one of the candidate reference-loops in $\mathscr{R}$. In practice, this is accomplished by monitoring test functionals of the form

$$V_i(t) = \mathfrak{V}_i(z^t, C_i, M_i) := \max\left\{ \frac{\|(\Delta_{*/i}[\xi_i])^\tau\|}{\|\xi_i^\tau\|}, \ \ \tau = 0, \cdots, t \right\}, \qquad i \in \underline{N} \tag{1.8}$$

where the vector-valued variables $\eta_i(t) := \Delta_{*/i}[\xi_i](t)$ and $\xi_i(t)$ are suitably selected in such a way that the upper-bound to (1.8) given by the induced gain $\|\Delta_{*/i}\|_{\text{ind}}$ of the transformation $\Delta_{*/i}$ from ξ_i to η_i, provides a measure of the distance between (P/C_i) and the i-th reference-loop (M_i/C_i). As will be shown, this implies that it be possible to effectively assess the suitability of C_i as a feedback controller for P from the magnitude of $V_i(t)$ which gives an estimate from below of $\|\Delta_{*/i}\|_{\text{ind}}$.

In Section 3.2, a manifold of alternative multi-model based UASC schemes will be considered, each one equipped with a different test functional of the form (1.8). Their features will be comparatively analyzed mainly in terms of generality of applicability, stability inference and on-line

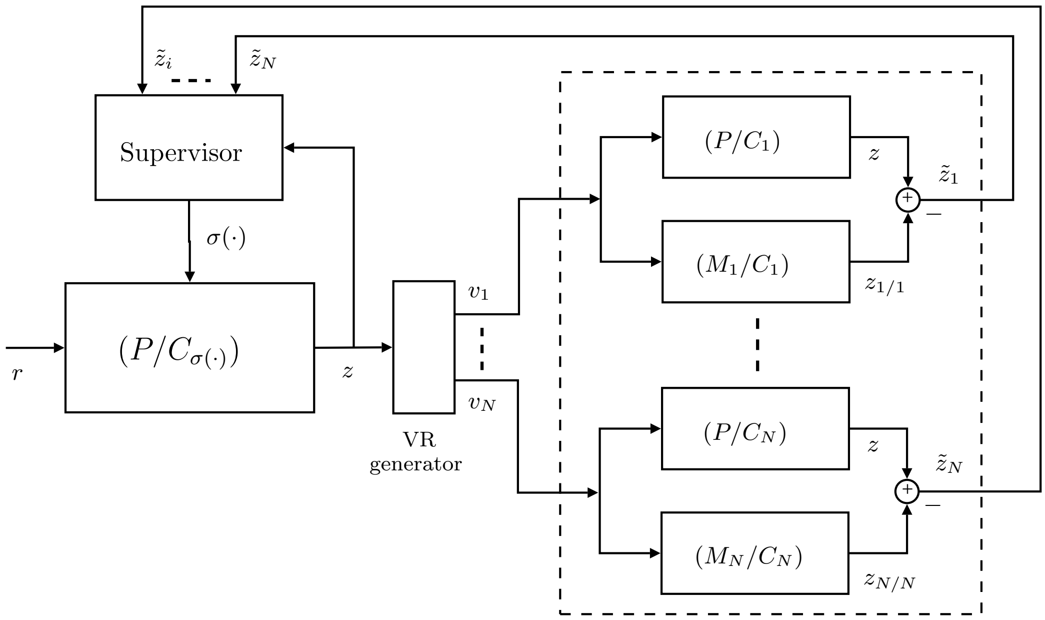


Figure 1.3: Block diagram of UASC for reference-loop identification.

implementability. It will be shown that, the test functional V_i in (3.35), which stems from considering robustness against the *coprime factor* uncertainty on P relatively to M_i, is the only one whereby the positive features of UASC and MMASC can coexist. In particular, it will be shown that multi-model UASC equipped with (3.35) not only enjoys model-free guaranteed stability properties (Lemma 3.3.1), but, in accordance with Theorem 3.2.2, also moderates $-$ in a *quantitative* sense $-$ the chance that destabilizing controllers be switched-on in case noises/disturbances are zero and the plant P in (1.3) belong to a *parametric* uncertainty set $\mathscr{P} := \{\, P(\theta),\ \theta \in \Theta \,\}$, consisting of a family of strictly causal LTI plants

$$P(\theta): \quad a(d,\theta)\, y(t) = B(d,\theta)\,(u(t) + n_u(t)) + a(d,\theta)\, n_y(t), \qquad t \in \mathbb{Z}_+ \tag{1.9}$$

where $a(d,\theta)$ and $B(d,\theta)$ denote polynomials with strictly Hurwitz g.c.d. In connection with the results of Theorem 3.2.2, Proposition 3.2.4 provides sufficient conditions under which the family $\mathscr{R}$ of reference-loops can be construced so as to satisfy the validity assumptions of Theorem 3.2.2.

1.1.3 Multi-model UASC implementation

Chapter 4 describes relevant implementational issues and extends the conclusions derived in Chapter 3 to a noisy environment. Section 4.1 exhibits efficient on-line computational architectures for general initial conditions. In particular, Theorem 4.1.1 shows that, even in case of nonzero plant initial conditions, it is possible to ensure that multi-model UASC systems equipped with (3.35) enjoy properties similar to the ones stated in Theorem 3.2.2. It will be also shown that the proposed scheme, though conceptually hinging upon the virtual reference entity (Fig. 1.3), is on-line implementable without an explicit v_i-computation. This is an advantageous feature from an implementational viewpoint, as computation of (1.4) is numerically impossible in case of non-stably invertible controllers. As it emerges from the analysis, an interesting feature of multi-model UASC is that its implementation can be carried out by suitably filtering the prediction errors ϵ_i's,

$$\epsilon_i(t) := a_i(d)y(t) - B_i(d)u(t), \qquad t \in \mathbb{Z}_+ \tag{1.10}$$

based on the nominal models M_i's. This establishes a conceptual link between the present approach and earlier MMASC contributions, since the multi-estimator of [Mor95, Mor96, NB97, HLM$^+$01], in a discrete-time setting, reduces to compute stably-filtered prediction errors. Motivated by such a fact, in Section 4.1, we provide a more detailed comparison between the proposed scheme and MMASC as well as an interpretation of the filtering action used in multi-model UASC in terms of frequency-domain analysis and connections with Identification for Control [Gev93, dHS95, MA01].

Finally, Section 4.2 further extends the stability results of multi-model UASC to a noisy environment. In this respect, Corollary 4.2.1, along with Theorem 4.1.1, complete the extension of the model-free case analysis of Chapter 2 to the present model-based approach to UASC. For illustrative purposes, in Section 4.3, the results are applied in the two carts example of [GNL95].

1.2 Horizon-switching Predictive Control of Linear Plants subject to Input Constraints

The main problem around which Part II of this thesis is centered is the control of critically unstable linear plants [6] subject to input constraints and persistent disturbances. The study of input-constrained systems is an extremely relevant issue in control engineering, since, without explicitly accounting for possible actuators limitations, decay of performance and/or stability loss due to delay and phase-lag might be experienced. Control of input-constrained plants has been given exhaustive and constructive systematic answers mainly only during the last twenty years. From one side it was characterized by the class of dynamical linear time-invariant (LTI) systems whose state can be asymptotically driven to zero with arbitrarily small controls [SSY94, Bla99, HL01b]: the so called ANCBI (Asymptotically Null-Controllable with Bounded Input) systems. From another side, linear control structures were shown to only provide semi-global stabilization of input-saturated ANCBI systems [Lin95, Lin98]. Non-linear state feedback schemes for input-saturated ANCBI plants were discussed in [SY91, SSY94]. A third source of contributions to the topic has been originated within

[6] In discrete-time, they coincide with LTI systems with eigenvalues in the closed unit disk.

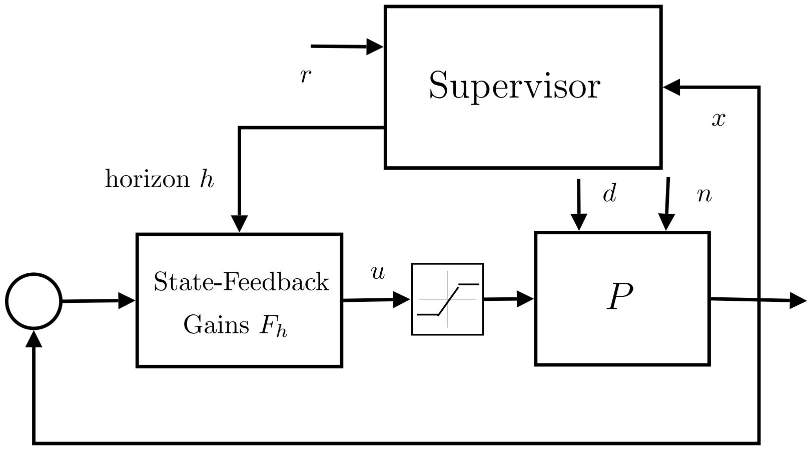


Figure 1.4: Horizon-switching predictive control scheme.

both model-based predictive control (MBPC) [GPM89, Mos95, MRRS00, GGS01] and anti-windup control schemes [HKH87, Tee92, GOMW00, GHP$^+$03]. Despite the vast literature on the subject, performance and/or stability issues in the presence of persistent arbitrarily bounded disturbances has often been disregarded, or the proposed solutions appear computationally demanding [SM98]. In a recent paper [Mos05], a computationally affordable solution to the pure regulation problem of discrete-time input-saturated LTI systems subject to persistent bounded disturbances of unknown arbitrary magnitude has been proposed.

Part II of this thesis aims at extending the approach of [Mos05] to the case of set-point tracking for LTI systems subject to joint positional and incremental input saturations and also affected by persistent disturbances of unknown arbitrary magnitude. It is worth saying that, while positional input saturations have attracted a great deal of interest, fewer results apply to incremental input saturations. Incremental input saturations are a serious challenge in many automatic control applications [Dor92, Len90, LPBS97, TB97, Lin98] as they can induce significant destabilizing effects due to delay and phase-lag [BHSB96]. As in [Mos05], the solution here proposed is realized via a supervisory switching control scheme whereby a feedback-gain, selected from a finite family of pre-designed candidate feedback-gains, is at any time switched-on in feedback to the plant according to the previous feedback-gain and the information, either complete or partial, on the current plant state (Fig. 1.4). As it will be shown throughout Chapter 5 and Chapter 6, the proposed scheme enjoys the following features: 1) The feedback-gain selection is made in accordance with a predictive control philosophy, and each candidate feedback-gain is tuned on to a different control-horizon; 2) No disturbance upper-bound need to be known; 3) The supervisory switching logic is flexible enough so as to enable the designer to simplify the scheme by trading off performance vs. memory and/or computational complexity, while retaining guaranteed stability properties.

1.2.1 Feasibility of horizon-switching predictive control

Consider the following discrete-time LTI ANCBI system

$$\begin{cases} x(t+1) &= \Phi x(t) + Gu(t) + \xi(t) \\ y(t) &= Hx(t) + \zeta(t) \end{cases} \tag{1.11}$$

with (Φ, G) reachable; Φ has all its eigenvalues of modulus less than or equal to one with arbitrary multiplicities; state $x \in \mathbb{R}^{n_x}$; input $u \in \mathbb{R}^m$; output $y \in \mathbb{R}^m$; ξ and ζ are exogenous disturbances. The plant input $u(t)$ and its increments $\delta u(t) := u(t) - u(t-1)$, $t \in \mathbb{Z}_+$ are subject to the following saturation constraints

$$u(t) \in \mathscr{U} := \{u \in \mathbb{R}^m : \quad |u|_i < \overline{U}\} \tag{1.12}$$

$$\delta u(t) \in \mathscr{D} := \{\delta u \in \mathbb{R}^m : \quad |\delta u|_i < \overline{\Delta}\} \tag{1.13}$$

where $i \in \underline{m}$, $|u|_i$ and $|\delta u|_i$ denote the absolute value of the $i-th$ component of u and, respectively, δu. The problem is to find, based on x (or a partial state-information), feedback controls

$$u = f(x) \tag{1.14}$$

which ensure global asymptotic stability and offset-free tracking in the presence of constant disturbances and set-point r, as well as finite l_∞-induced gain of the disturbance-to-state map from ξ to x embedded in (1.11)-(1.14) in the presence of time-varying disturbances and set-point. The adopted approach consists of selecting a discrete family $\mathscr{F} = \{F_h\}_{h=1}^{\infty}$ of linear state-feedback gains F_h and a switching logic

$$h(t) = \ell(x(t), h(t-1)) \tag{1.15}$$

in such a way that the regulated plant have the stated stability properties. In the proposed solution, each candidate feedback-gain is tuned on to a different control-horizon. More precisely, each F_h is taken to be the receding horizon regulator associated to the zero terminal-state minimum energy control problem of horizon h [KP75, Mos95]. The main result of Chapter 5 is the statement referred to as the *Feasibility Property*: It states that, in the presence of constant disturbances ξ, ζ and set-point r, jointly within the input control range, it is always possible to find a large enough control horizon h such that the corresponding control action (1.14) does not violate the input constraints (1.12) and (1.13). Taking into account the Feasibility Property, the question to be posed is how to properly select (1.15) so as to ensure stability and offset-free tracking (possibly with extra performance features) for the closed-loop switched system. As discussed in Theorem 5.3.1 of Section 5.3.2, in the absence of disturbances and input constraints, the ANCBI system (1.11) remains stable under arbitrary *admissible* switching, *viz.* any arbitrary switching sequence $\{h(t)\}_{t\in\mathbb{Z}_+}$, $h(t) \in \mathbb{Z}_+$, $h(t) \geq n_x + m$, subject to the following constraint [7]

$$h(t+1) \geq h(t) - 1 \tag{1.16}$$

The condition (1.16), first appeared in [Mos05], will be used in Chapter 6, along with the Feasibility Property, to prove that stability of ANCBI systems as in (1.11) also holds under input constraints and persistent disturbances.

1.2.2 Supervisory control under input constraints and persistent disturbances

In accordance with the internal model principle [Mos95, GGS01], a classic approach to track a reference r is to enforce an "integral action" from $e := y - r$ to u, r being the output reference. The related design hinges upon the so-called *incremental model* (IM) resulting from (1.11)

$$\begin{cases} \chi(t+1) &= \mathcal{A}\,\chi(t) + \mathcal{B}\,\delta u(t) + \delta v(t) \\ e(t) &= \mathcal{C}\,\chi(t) + \delta w(t) \end{cases} \tag{1.17}$$

where $\chi(t) := [\delta x'(t) \quad e'(t-1)]'$, $\delta x(t) := x(t) - x(t-1)$, $\delta v(t) := [\delta\xi'(t) \quad \delta w'(t)]'$, $\delta w(t) := \delta\zeta(t) - \delta r(t)$. It is well known that a linear state-feedback law $\delta u(t) = \mathscr{F}\chi(t)$, which stabilizes (1.17), yields an offset-free steady-state tracking error for the class of constant disturbances and references. Accordingly, the family $\mathscr{F} = \{F_h\}_{h=1}^{\infty}$ of linear state-feedback gains introduced in Chapter 5 is computed by solving the zero terminal-state minimum energy control problem of horizon h relatively to the model (1.17). This means that, for each control horizon h, one computes the sequence

$$\delta u_h(\chi) := [\delta u_h'(0|\chi), \ldots, \delta u_h'(h-1|\chi)]' = [\mathscr{F}_h'(0) \cdots \mathscr{F}_h'(h-1)]'\chi = \mathscr{F}_h\chi \tag{1.18}$$

which drives the system state χ to the zero-state 0_χ in h time-steps with minimum control effort $\sum_{k=0}^{h-1} \delta u'(k)\,\delta u(k)$; then, once the horizon $h(t)$ has been selected, the control law for the system (1.11) is given by

$$u(t) := u(t-1) + \mathscr{F}_{h(t)}(0)\chi =: F_{h(t)}\chi \tag{1.19}$$

viz. one applies the first component of the sequence (1.18) with $h = h(t)$, as in the standard receding horizon regulation problem [Mos95]. Based on the Feasibility Property and the admissibility condition (1.16) of Chapter 5, the hysteresis switching logic (HSL) is chosen as follows

$$h(t) = \begin{cases} \tilde{h}(t), & \text{if } \mathcal{M}_{\tilde{h}(t)}(\chi(t)) < 1 \\ \hat{h}(t), & \text{otherwise.} \end{cases} \tag{1.20}$$

[7] In connection with (1.16), it is worth noting that the problem of finding rules capable of ensuring stability under arbitrary switching sequences has recently attracted a great deal of interest [DM99, HM02].

$$\tilde{h}(t) := \max\{\underline{h},\ h(t-1)-1\}$$

$$\hat{h}(t) := \min\{h \in \mathbb{Z}_+ : \quad h \geq h(t-1); \quad \mathcal{M}_h(\chi(t)) \leq 1-\mu\}$$

where $\mu \in (0,\ 1)$ is the *hysteresis constant*; $h(0) = \hat{h}(0)$; $\underline{h}$ depends on the reachability index of (1.11) and $\mathcal{M}_h(\chi)$ is defined as

$$\mathcal{M}_h(\chi) := \max\left\{\tilde{\alpha}\frac{|[\delta u_h(k|\chi)]_i|}{\overline{\Delta}}\ ,\ \alpha\frac{|[u_h(k|\chi)]_i|}{\overline{U}}, \quad k+1 \in \underline{h},\ i \in \underline{m}\right\}, \tag{1.21}$$

where $(\tilde{\alpha} = 1, \alpha = 0)$ corresponds to only incremental saturations; $(\tilde{\alpha} = 0, \alpha = 1)$ pertains to only positional saturations; $(\tilde{\alpha} = 1, \alpha = 1)$ to joint incremental and positional saturations. In practice, the supervisory switching control scheme resulting from (1.20) selects, at every time instant, the maximum regulation speed compatible with the saturation constraints.

Theorem 6.2.1 in Section 6.2 shows that, in the presence of constant disturbances ξ, ζ and set-point r, jointly within the input control range, the HSL (1.20) yields offset-free asymptotic tracking for any possible pair $(\tilde{\alpha}, \alpha)$ in (1.20). Section 6.3 extends the stability analysis to the case of time-varying disturbances and set-point. In particular, Theorem 6.3.1 shows that the switched system resulting from (1.20) has finite l_∞ induced gain of the disturbance-to-state map in the presence of time-varying disturbances and set-point, while preserving the fulfillment of control-increment saturation constraints, *viz.* $\alpha = 0$ in (1.20). In this respect, the main contribution of Chapter 6 is indeed the proof that all arbitrary bounded disturbances, that can be in principle tolerated by (1.11) under positional input saturations for the pure regulation problem, can also be effectively handled under input-increment saturations, even when the output that is used for tracking is affected by arbitrary bounded disturbances.

For illustrative purposes, in Section 6.4, the results are applied to the control of the roll angle of an aircraft [Veg94] as well as in the two carts example of [GNL95]. Finally, in Appendix B, we show that there are some properties of the feedback-gains F_h's which can be conveniently exploited for keeping the memory/computational load requirements of (1.20) at a moderate level.

Part I

Multi-Model Unfalsified Adaptive Switching Control

Chapter 2

On Stability of Unfalsified Adaptive Switching Control

Among many approaches to adaptive switching control (ASC) − [Mor95] and [Lib03] provide an overview of the topic − a convenient framework to work with is the *unfalsified* ASC (UASC, for short) of [ST95, ST97]. In UASC, thanks to the use of the virtual reference tool, the scheduling task (when to substitute the acting controller) and routing task (which controller to switch on) are carried out without switching on all candidate controllers in the feedback loop in order to assess their performance with the plant. On the contrary, using recorded plant I/O data, the supervisor infers the performance of each potential loop and selects the one with the best inferred performance. To date, little attention has been devoted to the applicability of UASC in case the measurement I/O data be corrupted by noise. Theorem 2.2.1 in Section 2.2 extends the stability results of [SWPS07] to arbitrary signal norms in order to take into account the presence of noises/disturbances. In connection with Theorem 2.2.1, Section 2.3 shows that in UASC, stability results are restricted to the switched system, and no claim on stability of the unswitched final closed-loop can be advanced. For this reason, mixed-norm test functionals are finally introduced in Section 2.4 in order to ensure that the switched system be jointly input-output stable with respect to different norms.

2.1 Background

Let $\mathbb{S}$ denote the linear space of all the real-valued sequences on $\mathbb{Z}_+$. Given a vector-valued sequence $x \in \mathbb{S}$ of dimension n, x^t denotes its time truncation up to time t, *i.e.*, $x^t := \{x(0), x(1), \ldots, x(t)\}$, with $x(k) \in \mathbb{R}^n$. Let $\ell_p(\mathbb{Z}_+)$, $p \geq 1$, be the space of all vector-valued real sequences with bounded ℓ_p-norm defined as

$$\|x\|_p^p := \sum_{k=0}^{\infty} |x(k)|_p^p = \sum_{k=0}^{\infty}\sum_{i=1}^{n} |x_i(k)|^p, \quad \|x\|_\infty := \sup_k \max_i |x_i(k)| \tag{2.1}$$

where $|\cdot|$ denotes Euclidean norm. *E.g.*, $l_2(\mathbb{Z}_+)$ and $l_\infty(\mathbb{Z}_+)$ denote the linear spaces of sequences belonging to $\mathbb{S}$ with bounded energy, respectively, peak. Another important class of signals is finite-power signals. In such a case, we denote with $\mathcal{P}(\mathbb{Z}_+)$ the linear space consisting of those sequences $x \in \mathbb{S}$ with bounded $\mathcal{P}$-norm defined as

$$\|x\|_{\mathcal{P}}^2 := \sup_t \frac{1}{t+1} \sum_{k=0}^{t} |x(k)|_2^2 = \sup_t \frac{1}{t+1} \sum_{k=0}^{t}\sum_{i=1}^{n} |x_i(k)|^2 \tag{2.2}$$

Hereafter, $\|\cdot\|$ will denote a generic signal norm among the ones defined in (2.1) and (2.2), and $\mathcal{N}(\mathbb{Z}_+)$ its related normed space [1] . The following notion of input-output stability is adopted:

Definition 2.1.1 – A causal system H with input $w = \{w_i,\, i \in \underline{K}\}$, $\underline{K} := \{1, \cdots, K\}$, and output $z = \{z_j,\, j \in \underline{M}\}$ is said input-output $\mathcal{N}$-stable if, for every $w \in \mathbb{S}$, there exist finite nonnegative constants α, β such that

$$\|z^t\| \leq \alpha\, \|w^t\| + \beta, \qquad t \in \mathbb{Z}_+ \tag{2.3}$$

where z denotes the system output response to the input w.

The constant β allows for consideration of systems with non-zero initial state. It should be emphasized that stability of the system H requires that (2.3) holds true, possibly with different constants α, β, for any possible input. In UASC, unfalsified stability is related to a single infinite-length experiment, where some of the inputs (*e.g.*, disturbances) are inaccessible. In this connection, supposing that the first L inputs are accessible, and the inaccessible inputs $\{w_i,\, i = L+1, \ldots, K\}$ belong to $\mathcal{N}(\mathbb{Z}_+)$, the following definition is crucial for the subsequent developments.

Definition 2.1.2 – Given a system H with input $w = \{w_i,\, i \in \underline{L}\}$ and output $z = \{z_j,\, j \in \underline{M}\}$, $\mathcal{N}$-stability of H is said to be unfalsified by a specific input-output pair $\{w,\, z\}$ if there exist finite nonnegative constants γ, δ such that

$$\|z^t\| \leq \gamma\, \|w^t\| + \delta, \qquad t \in \mathbb{Z}_+ \tag{2.4}$$

In (2.4), the constant δ accounts for both the non-zero initial state and the inaccessible inputs. Of course, unfalsified stability can be used to empirically test stability only when the disturbances are known to have bounded $\mathcal{N}$-norm.

[1] As shown in [BMST09], the results which follow can be easily extended to any signal norm $\mathfrak{N}$ associating a nonnegative real $\|s^t\|$, $t \in \mathbb{Z}_+$, to each truncated sequence s^t and satisfying the following assumption:

(a) if the sequence s converges exponentially to zero then $\|s^t\|$ is bounded, i.e., there exists a nonnegative finite real α such that $\|s^t\| \leq \alpha$ for any $t \in \mathbb{Z}_+$.

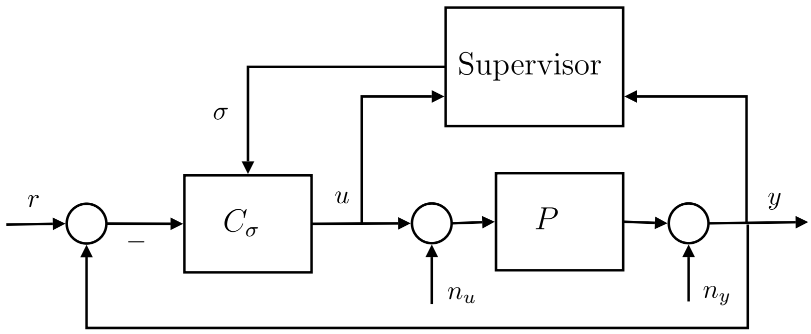


Figure 2.1: Typical ASC arrangement.

2.2 Switched System Stability in a Noisy Environment

The study is intentionally focused on the simplest nontrivial case of a strictly causal SISO plant consisting of a discrete-time time-invariant dynamic system $P : \mathbb{S} \to \mathbb{S}$. Along with the uncertain plant P, a finite family $\mathscr{C} = \{C_i, i \in \underline{N}\}$ of one-degree-of-freedom LTI controllers C_i is available. Each candidate C_i is described by a difference equation of the form

$$C_i : \quad r_i(d)u(t) = S_i(d)(r(t) - y(t)), \qquad t \in \mathbb{Z}_+ \tag{2.5}$$

with $r_i(d) = 1+\sum_{n=1}^{p} r_{in}d^n$ and $S_i(d) = s_{i0}+\sum_{n=1}^{p} s_{in}d^n$ coprime polynomials in the unit backward shift operator d, with p denoting the maximum order among all controllers in $\mathscr{C}$. Notice that $r_i(0) = 1$ ensures that C_i be causal. Thus, by assuming that the input and the output of the plant are affected be unknown additive disturbances n_u and n_y, the ASC system (Fig. 2.1), denoted by $(P/C_{\sigma(\cdot)})$, is as follows [2]

$$\begin{cases} y(t) = P[u + n_u](t) + n_y(t) \\ u(t) = C_{\sigma(t)}[r - y](t) \end{cases}, \qquad t \in \mathbb{Z}_+ \tag{2.6}$$

where $\sigma(t)$, $\sigma(t) \in \underline{N}$, is the index that identifies the controller connected in feedback to P at time t. Given a family $\mathscr{C}$ of candidate controllers, the goal of the supervisory unit is selecting $\sigma(t)$ in (2.6) so as so ensure that $(P/C_{\sigma(\cdot)})$ be $\mathcal{N}$-stable, *viz.* ensuring (2.3) with $z := \{u, y\}$ and $w := \{r, n_u, n_y\}$. Accordingly, the minimal requirement on $\mathscr{C}$ is as follows.

Problem Feasibility 1: The candidate controller set $\mathscr{C}$ contains at least one controller C_i such that (P/C_i) is $\mathcal{N}$-stable.

[2]Hereafter, $v = \mathfrak{P}[u]$ denotes a causal mapping from the input vector u to the output vector v.

The state of a controller at the switch-on time can be either arbitrarily initialized or, perhaps, initialized by bumpless control transfer techniques [GGS01] or common state multicontroller schemes [Mor95], so as to reduce as much as possible the transients caused by switching from one controller to a different one, particularly in case some candidate controllers are open-loop unstable (see Appendix A).

The high-level device responsible for orchestrating $\sigma(t)$ is the switching supervisor. As anticipated, in UASC, the feedback adaptation task of classic adaptive control is replaced by controller falsification. The active controller can be falsified via a comparative experiment, by resorting to the *virtual* reference concept introduced in [ST95]. To this end, for each $i \in \underline{N}$, one solves in real-time the difference equation

$$r_i(d)u(t) = S_i(d)(v_i(t) - y(t)), \qquad t \in \mathbb{Z}_+ \tag{2.7}$$

with respect to the virtual reference v_i, which can be done provided that C_i is causal, and stably causally invertible (CSCI), that is, $S_i(d)$ strictly Hurwitz. In words, v_i equals the fictitious reference that, if injected into the feedback system (P/C_i), would reproduce $z = \{u, y\}$. In other terms, if $(P/C_{\sigma(\cdot)})$ is intended as the (time-varying) causal transformation (2.6) mapping $w = \{r, n_u, n_y\}$ into z, we find:

$$z := (P/C_{\sigma(\cdot)})[w] = (P/C_i)[w_i] \quad \Leftrightarrow \quad \begin{cases} y(t) = P[u + n_u](t) + n_y(t) \\ u(t) = C_i[v_i - y](t) \end{cases}, \ t \in \mathbb{Z}_+ \tag{2.8}$$

where $w_i := \{v_i, n_u, n_y\}$ and (P/C_i) is the (time-invariant) causal transformation made up by the plant P fed-back by C_i.

Remark 2.2.1 – While next results hinge upon the assumption that all C_i's be CSCI, one can prove, along the same lines as in [MCMS07], that the same conclusions hold true for possibly non-stably invertible C_i's provided that a modified v_i, call it ϖ_i, *e.g.*, $\varpi_i(t) = r_i(d)u(t) + S_i(d)y(t)$, be appropriately used. Furthermore, the restriction according to which all the controllers are LTI can be removed [SWPS07].

The introduction of the v_i's makes it possible to causally compute from z nonnegative *test functionals* $V_i(t) := \mathfrak{V}_i(z^t)$, $t \in \mathbb{Z}_+$, pairwise associated with the candidate controller C_i. At every $t \in \mathbb{Z}_+$, the supervisor compares the N test functionals and selects the controller index via the following *Hysteresis Switching Logic* (HSL) [3]:

$$\sigma(t+1) \in \arg\min_{i \in \underline{N}} \left\{ V_i(t) - h\, \delta_{i,\sigma(t)} \right\}, \quad \sigma(0) = \sigma_0, \tag{2.9}$$

where σ_0 is arbitrary in $\underline{N}$, $h > 0$ is the hysteresis constant and $\delta_{i,j}$ the Kronecker's index. Whenever arg min is not unique, a particular value for $\sigma(t+1)$ is chosen among those that achieve the minimum.

[3] Alternatively to (2.9), a multiplicative HSL $\sigma(t+1) \in \arg\min_{i \in \underline{N}} \left\{ V_i(t)(1 - h\, \delta_{i,\sigma(t)}) \right\}$, $0 < h < 1$, – the so-called *scale-independent HSL* [Hes98, HLM+01] – can be considered.

2.2.1 Main result

In UASC analysis, a fundamental role is played by the next Hysteresis Switching Logic Lemma [MMG92]. Let $\mathfrak{S}$ denote the class of all possible switching functions $\sigma : \mathbb{Z}_+ \to \underline{N}$ giving rise to the switched system $(P/C_{\sigma(\cdot)})$. Consider the assumptions:

A1. For each $s \in \mathfrak{S}$ and $i \in \underline{N}$, $V_i(t)$ admits a limit (even infinite) as $t \to \infty$;

A2. There is at least one integer $m \in \underline{N}$ such that $V_m(\cdot)$ is bounded for each $s \in \mathfrak{S}$.

Lemma 2.2.1 (**HSL Lemma**) – Consider the ASC system (2.6) with $\sigma(t)$ selected in accordance with the HSL (2.9). Then, if A1 and A2 hold, for any initial condition and any reference r, there is a finite time t_f such that $\sigma(t) = f$ for every $t \geq t_f$. Moreover, $V_f(\cdot)$ is bounded. $\square$

The key property enabling test functionals to be used in the HSL Lemma relies on the concept of cost-detectability, here adapted from [SWPS07].

Definition 2.2.1 – Given the test functional $V := \{V_i,\ i \in \underline{N}\}$ and the controller family $\mathscr{C}$, the pair $(V, \mathscr{C})$ is said to be $\mathcal{N}$-*cost-detectable* if for every $C_{\sigma(\cdot)} \in \mathscr{C}$ with finitely many switching times and final switched-on controller $C_f \in \mathscr{C}$, the following statements are equivalent: 1) $V_f(\cdot)$ is bounded; 2) Stability of the system $(P/C_{\sigma(\cdot)})$ mapping r into z is unfalsified by $\{r, z\}$.

Cost detectability [4] allows one to use suitable V_i's for detecting any instability trend exhibited by the ASC system (2.6). To see this, it is convenient to avail of the following lemma.

Lemma 2.2.2 – Consider the ASC system (2.6) with $\sigma(t)$ selected in accordance with the HSL (2.9). Suppose that controller switching eventually stops and that the final controller C_f is CSCI. Then the virtual reference v_f converges exponentially to the true reference r.

Proof. For the final controller C_f, equation (2.7) yields $S_f(d)v_f(t) = S_f(d)y(t) + r_f(d)u(t)$. Further, regardless of the state the controller C_f is in at time t_f, one has that for $t > t_f + p$ with $p = \max\{\deg S_f, \deg r_f\}$

$$r_f(d)u(t) = S_f(d)r(t) - S_f(d)y(t)$$

Thus, one concludes that $S_f(d)(v_f(t) - r(t)) = 0$, $t > t_f + p$. This, in turn, implies that $v_f(t) - r(t)$ converges exponentially to zero being $S_f(d)$ strictly Hurwitz by hypothesis. $\square$

In the light of Lemma 2.2.2, a simple way to enforce cost-detectability is to select V so as to provide an estimate of the $\mathcal{N} - \mathcal{N}$ induced gain of the map embodied by (P/C_i) from v_i to z, *e.g.*

$$V_i(t) := \max\left\{ \frac{\|z^\tau\|}{\mu + \|v_i^\tau\|},\ \ \tau = 0, \cdots, t \right\} \tag{2.10}$$

[4] Note that the notion of Cost Detectability coincides with the concept of Gain-Relatedness in case, as the present one, the controllers are LTI and CSCI [SWPS07].

where $\mu > 0$ accounts for possible non-zero initial states as well as unknown bounded disturbances. Functionals as (2.10), in the context of the l_2-norm and noise-free environments, have been widely analyzed [SWPS07, SS08]. Next theorem extends such results to arbitrary signal norms as well as to the presence of unknown exogenous disturbances.

Theorem 2.2.1 – *Consider the ASC system (2.6) with $\sigma(t)$ selected in accordance with the HSL (2.9). Let all the LTI candidate controllers be CSCI. Then, for any possible r, provided that:*

B1. *The pair $(V, \mathscr{C})$ is $\mathcal{N}$-cost-detectable;*

B2. *Problem Feasibility 1 holds; and*

B3. *The disturbances n_u and n_y belong to $\mathcal{N}(\mathbb{Z}_+)$;*

the HSL Lemma holds and $\mathcal{N}$-stability of the final feedback system (P/C_f) is unfalsified by the experimental pair $\{r, z\}$. Consequently, the ASC system is $\mathcal{N}$-stable.

Proof. See Appendix A. □

The assumption that the disturbances n_u and n_y belong to $\mathcal{N}(\mathbb{Z}_+)$ is crucial to ensure that the test functionals V_i's remain bounded for the indices related to $\mathcal{N}$-stabilizing controllers. An important consequence of this state of affairs is that test functionals defined with respect to the empirical l_2-gain are inadequate to deal with disturbances belonging to $l_\infty(\mathbb{Z}_+)$ but not to $l_2(\mathbb{Z}_+)$. The interested reader is referred to [BMST08] for an in-depth discussion of this issue.

2.3 Relationships among Stability Notions in Unfalsified ASC

As a consequence of Theorem 2.2.1, $\mathcal{N}$-cost-detectability of $(V, \mathscr{C})$ allows one to select a time-varying control yielding an $\mathcal{N}$-stable switched system $(P/C_{\sigma(\cdot)})$, provided that Problem Feasibility 1 hold. However, $\mathcal{N}$-stability of $(P/C_{\sigma(\cdot)})$ does not imply that the the final loop (P/C_f) be such. In fact, Theorem 2.2.1 does not state that the switching index $\sigma(t)$ stops onto an $\mathcal{N}$-stabilizing controller: In order to enforce such a property, an adequate excitation is indeed required. Theorem 2.2.1 only states that $\mathcal{N}$-stability of (P/C_f) is unfalsified by the experimental data $\{r, z\}$. Accordingly, even if P is a finite dimensional LTI (FDLTI) system (and, hence, (P/C_f) is such), one cannot resort to classic stability results [BD87] as $\mathcal{N}$-stability unfalsification of (P/C_f) by $\{r, z\}$ need not imply unfalsification of (P/C_f) by $\{r, z\}$ relatively to any other normed space deriving from (2.1) and (2.2). As exemplified below, Theorem 2.2.1 assumptions are indeed non-conservative and indicate that careful attention must always be paid to the test functional choice.

2.3.1 Example # 1

Let $P(d) := \alpha\, d$, $\alpha > 0$, be the transfer function of the uncertain plant, and

$$C_j(d) := \alpha\,/(\,1 - (1+\alpha^2)\,d) \tag{2.11}$$

be the transfer function of one of the candidate controllers in the family $\mathscr{C}$. It is immediate to see that C_j does not stabilize P. In fact, the transfer functions G_{ur} and G_{yr} of (P/C_j) which map r into u, and, respectively, y, are given by $G_{ur}(d) = \alpha/(1-d)$ and $G_{yr}(d) = \alpha^2 d/(1-d)$. Consider two functionals $V_j^{(p)}$ as in (2.10), computed with respect to the l_2-norm and the l_∞-norm.. For the sake of clarity, consider zero plant initial conditions and zero noises/disturbances. Suppose that C_j be initially switched-on. Accordingly, as long as C_j is kept in the loop, $v_j = r$, and

$$V_j^{(p)}(t) := \max\left\{ \frac{\|z^\tau\|_p}{\mu + \|r^\tau\|_p},\ \ \tau = 0, \cdots, t \right\}, \qquad p = 2, \infty \tag{2.12}$$

Suppose also that Problem Feasibility 1 be satisfied, and, in particular, that $\mathscr{C}$ contain a candidate controller which internally stabilizes P. Now, let $\sigma(t)$ be selected in accordance with the HSL (2.9) with $V_i(t) := V_i^{(2)}(t)$. As shown next, in response to certain reference signals, $V_j^{(2)}$ stays bounded over $\mathbb{Z}_+$. Thus, provided that the hysteresis h be sufficiently high, C_j cannot be switched-off. Since (P/C_j) is unstable, equivalence among I/O stability notions fails to hold. In particular, as $V_j^{(\infty)}$ grows unbounded, the switched system turns out to be l_∞-unstable. To see this, define

$$\mathfrak{A}(k) := 2\sum_{i=2}^{k}(i^3+i),\ \ \mathfrak{B}(k) := 2\left(\sum_{i=2}^{k}(i^3+i) + (k+1)^3\right)$$

and let $\mathbb{Z}_+ = \bigcup_{k\in\mathbb{Z}_1}(\,I_1^k \cup I_2^k)$, $\mathbb{Z}_1 := \{1, 2, \ldots\}$, where

$$I_1^k := \{\,\mathfrak{A}(k)\,, \cdots\,,\ \mathfrak{B}(k)\ - 1\,\}\,,$$

and $I_2^k = I_{2a}^k \cup I_{2b}^k$ is defined as

$$I_{2a}^k := \{\,\mathfrak{B}(k)\,, \cdots\,,\ \mathfrak{B}(k) + k\}\,,$$

$$I_{2b}^k := \{\,\mathfrak{B}(k) + k + 1\,, \cdots\,,\ \mathfrak{A}(k+1) - 1\,\}\,,$$

Consider now the following reference:

$$r(t) := \begin{cases} (-1)^t, & t \in I_1^k \\ \quad 1, & t \in I_{2a}^k, \qquad k \in \mathbb{Z}_1 \\ \ -1, & t \in I_{2b}^k \end{cases} \tag{2.13}$$

In words, over the interval I_2^k, the reference r is a rectangular wave of period increasing with k, while u and y are triangular waves with increasing amplitude. It is immediate to conclude that neither u nor y are in $l_\infty(\mathbb{Z}_+)$, then, $V_j^{(\infty)}$ defined in (2.12) grows to infinity. In fact, as depicted in Fig.2.2,

$$u(t) = \alpha\, n_k, \quad t \in I_{2a}^k \tag{2.14}$$

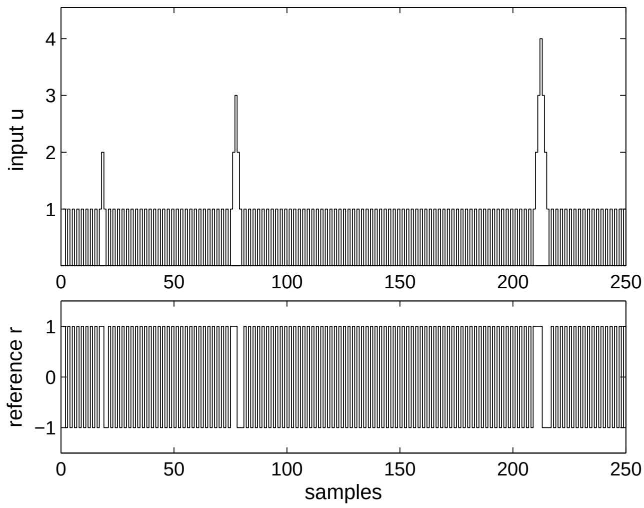


Figure 2.2: Reference r and control input u of Example # 1 for $\alpha = 1$

where $n_k \geq 1$, $k \in \mathbb{Z}_1$, is the n-th sample over I_{2a}^k. On the contrary, the functional $V_j^{(2)}$ defined in (2.12) stays bounded over $\mathbb{Z}_+$. To see this, notice first that $\|y^t\|_2^2 = \alpha^2 \sum_0^t u_2^2(k-1) \leq \alpha^2 \|u^t\|_2^2$, $t \in \mathbb{Z}_+$. Thus, it suffices to focus the attention on the ratio $\|u^t\|_2^2 \,/\, \|r^t\|_2^2$. Over the intervals I_2^i, $i \in \underline{k}$, $k \in \mathbb{Z}_1$, the input u is a triangular wave with energy $2\alpha^2 \sum_{j=1}^i j^2 + \alpha^2(i+1)^2$. Hence, over I_2^k, $k \in \mathbb{Z}_1$, one has

$$\begin{aligned} \|u^t\|_2^2 &\leq \alpha^2 \sum_{i=2}^{k+1} i^3 + \sum_{i=2}^{k+1} \left(2\alpha^2 \sum_{j=1}^{i-1} j^2 + \alpha^2 i^2 \right) \\ &= \alpha^2 \sum_{i=2}^{k+1} i^3 + \alpha^2 \sum_{i=2}^{k+1} \frac{2}{3} i^3 + \frac{1}{3} i \leq 2\alpha^2 \sum_{i=2}^{k+1} i^3 \end{aligned}$$

where the first term on the right hand side of the first inequality is the amount of the energy of u accumulated during I_1^k. Over the same intervals,

$$\|r^t\|_2^2 \geq 2 \sum_{i=2}^{k+1} i^3 \tag{2.15}$$

the latter being the amount of the energy of r accumulated during I_1^k. Hence,

$$\|u^t\|_2^2 \,/\, \|r^t\|_2^2 \leq \alpha^2, \qquad t \in I_2^k,\ k \in \mathbb{Z}_1 \tag{2.16}$$

On the opposite, over intervals I_1^k, $k \in \mathbb{Z}_1$, one finds

$$\|u^t\|_2^2 \leq 2\alpha^2 \sum_{i=2}^{k} i^3 + \alpha^2 n_k \tag{2.17}$$

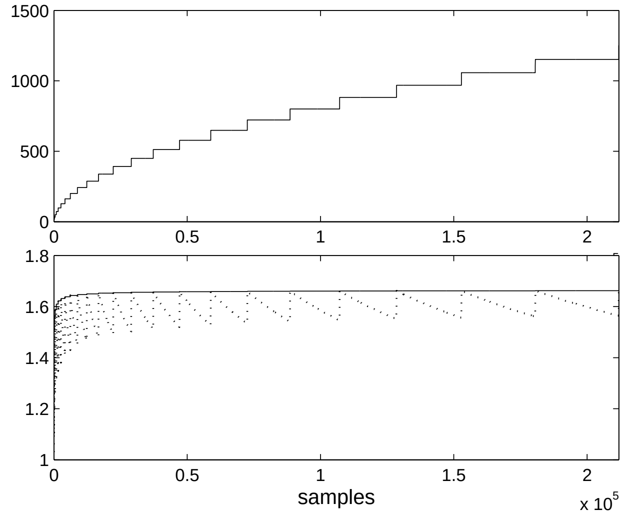


Figure 2.3: Behavior of the square of the functionals $V_j^{(\infty)}(t)$, and respectively, $V_j^{(2)}(t)$ $-$ solid line, dotted line without the maximum operator $-$ of Example # 1 for $\mu = 10^{-5}$ and $\alpha = 1$.

where the first term on the right hand side of the inequality is the amount of the energy of u accumulated during I_2^{k-1}, and $n_k \geq 1$, $k \in \mathbb{Z}_1$, is the n-th sample over I_1^k. Over the same intervals,

$$\|r^t\|_2^2 \geq 2 \sum_{i=2}^{k} i^3 + n_k \tag{2.18}$$

with n_k defined as beforehand. Hence,

$$\|u^t\|_2^2 \,/\, \|r^t\|_2^2 \leq \alpha^2, \qquad t \in I_1^k,\ k \in \mathbb{Z}_1 \tag{2.19}$$

Summing up and recalling that $\|y^t\|_2^2 \leq \alpha^2 \|u^t\|_2^2$, $t \in \mathbb{Z}_+$, one finds

$$V_j^{(2)}(t) \leq \frac{(1+\alpha^2)^{1/2}\,\|u^t\|_2}{\|r^t\|_2} \leq \alpha\,(1+\alpha^2)^{1/2}, \qquad t \in \mathbb{Z}_+ \tag{2.20}$$

Hence, the functional $V_j^{(2)}$ stays bounded, even if u and y are not in $l_\infty(\mathbb{Z}_+)$ (see Fig.2.3). Moreover, $V_j^{(2)}$ can be made as small as desired by decreasing α. This example also points out that arbitrarily large peak gains can be obtained with sequences r, u and $y \in l_2(\mathbb{Z}_+)$.

2.3.2 Example # 2

The fact that l_∞-stability unfalsification of (P/C_f) need not imply l_2-stability unfalsification of (P/C_f) is trivial. To see this, it suffices to consider the same system as in Example # 1 with the same controller initialization and with $r(0) = 1$, and $r(t) = 0$, $t \geq 1$.

2.4 Mixed-norm Test Functionals

As pointed out in the previous section, in general the notion of stability entailed by Theorem 2.2.1 is tied up to the particular norm adopted in the test functionals. Nevertheless, it turns out that adopting mixed-norm test functionals it is possible to ensure that the ASC system be jointly stable with respect to different norms. To see this, consider M different signal norms $\mathcal{N}^{(m)}, m \in \underline{M}$. Further, define, for each $i \in \underline{N}$, a mixed-norm test functional

$$V_i(t) = \sum_{m=1}^{M} \lambda^{(m)} V_i^{(m)}(t) \tag{2.21}$$

where each $V_i^{(m)}$ is an $\mathcal{N}^{(m)}$-cost-detectable test functional computable from z^t, and the $\lambda^{(m)}$'s are positive reals weights expressing the relative importance of the various functionals in (2.21). In order to ensure the mixed-norm control problem to be well-posed, the minimal requirement is as follows.

Problem Feasibility 2: The candidate controller set $\mathscr{C}$ contains at least one controller C_i such that (P/C_i) is stable with respect to all the norms $\mathcal{N}^{(m)}, m \in \underline{M}$.

Theorem 2.4.1 *Consider the ASC system (2.6). Let all the LTI candidate controllers be CSCI, and the HSL (2.9) used along with the test functionals V_i's as in (2.21). Then, for any possible r, provided that:*

C1. *For each $m \in \underline{M}$, the pair $(V^{(m)}, \mathscr{C})$ is $\mathcal{N}^{(m)}$-cost-detectable;*

C2. *Problem Feasibility 2 holds; and*

C3. *The disturbances n_u and n_y belong to*

$$\bigcap_{m=1}^{M} \mathcal{N}^{(m)}(\mathbb{Z}_+)\,,$$

the HSL Lemma holds and $\mathcal{N}^{(m)}$-stability, $m \in \underline{M}$, of the final feedback system (P/C_f) is unfalsified by the experimental pair $\{r,\, z\}$. Consequently, the ASC system is $\mathcal{N}^{(m)}$-stable, $m \in \underline{M}$.

Proof. – The proof can be given along the same lines of Theorem 2.2.1 and therefore omitted. The requirement that the noises belong to the intersection of the sets $\mathcal{N}^{(m)}(\mathbb{Z}_+)$ is needed to ensure that the mixed-norm test functional (2.21) remains bounded for indices yielding stabilizing controllers. □

Induced gains to which the functional is related	Reference in	Noises in	Stability achieved
l_2	$l_2(\mathbb{Z}_+)$	$l_2(\mathbb{Z}_+)$	l_2, l_∞, $\mathcal{P}$
l_2	$\mathbb{S}$	$l_2(\mathbb{Z}_+)$	l_2, $\mathcal{P}$
l_∞	$\mathbb{S}$	$l_\infty(\mathbb{Z}_+)$	l_∞
$\mathcal{P}$	$\mathbb{S}$	$\mathcal{P}(\mathbb{Z}_+)$	$\mathcal{P}$
l_2, l_∞	$\mathbb{S}$	$l_2(\mathbb{Z}_+)$	l_2, l_∞, $\mathcal{P}$
l_∞, $\mathcal{P}$	$\mathbb{S}$	$l_\infty(\mathbb{Z}_+)$	l_∞, $\mathcal{P}$

Table 2.1: Relationship between test functionals and stability

In the light of Theorem 2.4.1, it is immediate to see that $\mathcal{N}$-cost-detectable functionals of the form

$$V_i(t) = V_i^{(2)}(t) + \lambda\, V_i^{(\infty)}(t), \qquad i \in \underline{N}$$

yield an ASC systems that is stable with respect to both the l_2 and the l_∞-norms, provided that noises/disturbances belong to $l_2(\mathbb{Z}_+) = l_2(\mathbb{Z}_+) \cap l_\infty(\mathbb{Z}_+)$. If instead one has to deal with noises/disturbances belonging to $l_\infty(\mathbb{Z}_+)$, as $l_\infty(\mathbb{Z}_+) = l_\infty(\mathbb{Z}_+) \cap \mathcal{P}(\mathbb{Z}_+)$, by adopting test functionals of the form

$$V_i(t) = V_i^{(\mathcal{P})}(t) + \lambda\, V_i^{(\infty)}(t), \qquad i \in \underline{N}$$

it is possible to ensure that the resulting ASC system has finite l_∞ and $\mathcal{P}$-gains. Table 2.1 summarizes the most frequently used stability notions, which can be guaranteed by means of $\mathcal{N}$-cost-detectable test functionals.

2.4.1 Adoption of the exponentially weighted l_2-norm

In connection with Theorem 2.2.1, another relevant result can be achieved by means of the λ-exponentially weighted l_2-norm (shortly, $l_{2,\lambda}$-norm), defined as

$$\|x^t\|_{2,\lambda}^2 := \sum_{k=0}^{t} \lambda^{t-k}\, |x(k)|^2, \qquad \lambda \in (0,\, 1) \tag{2.22}$$

Along the same lines of Def. 2.1.2, $l_{2,\lambda}$-stability unfalsification of a candidate loop (P/C_i) by $\{v_i, z\}$ amounts to the existence of finite constants γ and δ such that

$$\|z^t\|_{2,\lambda} \leq \gamma\, \|v_i^t\|_{2,\lambda} + \delta, \qquad t \in \mathbb{Z}_+ \tag{2.23}$$

An important consequence of (2.23) is that test functionals defined with respect to the empirical $l_{2,\lambda}$-gain, such as

$$V_i(t) := \max\left\{ \frac{\|z^\tau\|_{2,\lambda}}{\mu + \|v_i^\tau\|_{2,\lambda}},\ \tau = 0, \cdots, t \right\}, \qquad \lambda \in (0,\, 1) \tag{2.24}$$

are an effective tool for dealing with noises/disturbances belonging to $l_\infty(\mathbb{Z}_+)$.

Corollary 2.4.1 Consider the ASC system (2.6). Let all the LTI candidate controllers be CSCI, and the HSL (2.9) used along with (2.24). Then, for any possible r, provided that:

D1 There exists at least one controller C_i such that (P/C_i) is $l_{2,\lambda}-$stable;

D2 The disturbances n_u and n_y belong to $l_\infty(\mathbb{Z}_+)$,

the HSL Lemma holds and the resulting ASC system is l_∞-stable.

Proof. – Under D1 and D2, there exists a candidate controller C_i such that, regardless of the switching sequence $\sigma(t)$, $t \in \mathbb{Z}_+$, (2.23) is valid. Consequently, the HSL Lemma holds. The rest of the proof can be given along the same lines as in Theorem 2.2.1. □

The rationale for distinguishing (2.24) from (2.10) stems from the fact that, with reference to *linear* plants, Problem Feasibility 1 and Problem Feasibility 2 are satisfied in case the candidate controller set $\mathscr{C}$ contains at least one controller, call it C_σ, such that (P/C_σ) is (internally) stable. In contrast with it, by assuming again plant linearity, $l_{2,\lambda}-$stability of (P/C_σ) (and, hence, safe operation with (2.24)) also requires that the spectral radius of (P/C_σ) be strictly smaller than $\sqrt{\lambda}$ [Dat93].

2.5 Concluding Remarks

Unfalsified Control of [ST95, Saf96] has a wide-ranging potential, beyond the one discussed in this chapter, *e.g.* on-line robust control design [KLB92, PKT+94, Kos95, ST97, BTS01], performance assessment of feedback control systems [TD07, IP08], and, more generally, all those control applications which involve problems in form of constrained optimization [DDB95].

In this chapter, attention has been mainly focused on the issue of generality of applicability of UASC in noisy environments. A closer examination of UASC stability reveals that, even in the simple case in which the plant P is FDLTI, one cannot obtain results of equivalence between different input-output stability notions, an issue this so far overlooked in the literature [DAL07, AD08], and deserving careful and detailed attention. The results presented in this chapter will serve as a basis for the developments of next chapters.

Chapter 3

Multi-Model Unfalsified Adaptive Switching Control

The main goal of Chapter 3 of this thesis is to develop novel highly effective multi-model UASC schemes based on the Unfalsified Control approach of Chapter 2.

Section 3.1 briefly recall the overall problem of interest, and adds complementary motivational interests to the developments elaborated in Section 3.2. The proposed scheme, called multi-model UASC, enjoys the following features: it moderates – in a *quantitative* sense – the chance that destabilizing controllers be switched-on in case noises/disturbances are zero and the plant uncertainty set $\mathscr{P}$ consists of a family of *linear* plants tightly approximated by a nominal model distribution $\mathscr{M}$; it also preserves the validity assumptions of Theorem 2.2.1 irrespective of both the actual plant and the nominal model distribution $\mathscr{M}$, *viz.* irrespective of possible plant-model mismatches caused by the introduction of prior assumptions on the plant, thus allowing application in case of large plant uncertainties. The results represent an important step towards the development of more reliable and effective ASC systems.

Notation – Given a causal linear operator $F[\cdot]$ with transfer matrix $F(d)$, and left matrix fraction representation $F(d) = G^{-1}(d)H(d)$ [Mos95], with input u and output y, $y(t) = F(d)u(t)$, by the notation $y_0(t) = (F)_0 [u](t)$ we mean that the sequence $y(t)$, $t \in \mathbb{Z}_+$, is computed via the following system of difference equations ($\det g_0 \neq 0$)

$$\sum_{k=0}^{n_G} g_k \, y(t-k) = \sum_{k=0}^{n_H} h_k \, u(t-k), \qquad y(k) = u(k) = 0, \quad k = -1, -2, \cdots \tag{3.1}$$

where $G(d) = \sum_{k=0}^{n_G} g_k \, d^k$ and $H(d) = \sum_{k=0}^{n_H} h_k \, d^k$. Further, $(F)_0$ is referred to as F from its zero initial conditions at time 0. $\square$

3.1 Overall Problem

The overall problem of interest is the one discussed in Chapter 2. Hereafter, since crucial to improving UASC performance − altough unnecessary for stability analysis (see Lemma 3.3.1) − it will be assumed that the plant P in (2.6) belong to a *parametric* uncertainty set $\mathscr{P} := \{ P(\theta),\ \theta \in \Theta \}$, consisting of a family of strictly causal finite-dimensional LTI plants

$$P: \ \ a(d,\theta)\, y(t) = B(d,\theta)\,(u(t) + n_u(t)) + a(d,\theta)\, n_y(t), \qquad t \in \mathbb{Z}_+ \tag{3.2}$$

where $a(d,\theta)$ and $B(d,\theta)$ denote polynomials with strictly Hurwitz g.c.d., and $a(d,\theta)$ monic, *viz.* $a(0,\theta) = 1$; strict causality of $P(\theta)$ amounts to $B(0,\theta) = 0$. For the sake of brevity, from now on, the parameter vector θ will be omitted in all polynomials, unless otherwise required for avoiding possible confusion.
In accordance with the mentioned restriction to linear plants, the minimal requirement on $\mathscr{C}$ is modified as follows.

Problem Feasibility: The candidate controller set $\mathscr{C}$ contains at least one controller C_i such that (P/C_i), $P \in \mathscr{P}$, is internally stable ($\Rightarrow \mathscr{N}-$stability, $\forall \mathscr{N}$).

3.1.1 Ideal goal of the switching supervisor

In connection with the ASC system $(P/C_{\sigma(\cdot)})$ in (2.6), the ideal goal of the switching supervisor can be envisaged as follows. Given an uncertain plant $P \in \mathscr{P}$, find an index $\sigma \in \underline{N}$ in such a way that:

(i) (P/C_σ) be (internally) stable;

(ii) The chance of switching on destabilizig controllers be minimized.

Loosely stated, an ASC scheme should be capable of providing rapid convergence to a final stabilizing (possibly with extra performance features) controller C_σ, *viz.* circumventing the major difficulties encountered in adaptive tuning schemes: long-lasting transients and danger of stabilizability loss of the identified model [Mos95, IS96]. In other terms, in connection with (i)-(ii), one should be able to promptly select C_σ (via switching), with the perspective that, over slower time scales, the controller parameters might be subsequently adjusted to improve accuracy (via tuning) [NB94, NB97]. Unfortunately, securing the stated goals is far from straightforward. As for (i), as noted in previous chapter, by restricting attention to deterministic algorithms that do not require persistency of excitation, it is conceptually impossible to ensure that the final closed-loop (if any) be stable [1]. As for (ii), ensuring that switching takes place only between stabilising controllers (the

[1] *E.g.*, the data may indeed supply no information at all, *viz.* $u(t) = y(t) = 0$, $t \in \mathbb{Z}_+$.

so-called *safe switching*) is a major theoretical problem. Attempts in this direction can be found in the ASC literature [ABLM01, DLLA09]. However, ruling out the possibility that a destabilising controller be switched-on, requires a high computational load, and typically calls for small controller changes, requirements these, in contrast with situations where the supervisor must promptly react in response to abrupt plant changes or wrong controller initializations.

Because of the mentioned difficulties, despite the vast literature on the subject (*e.g.*, see the survey paper [AD08]), no single approach has so far distinctly encountered unanimous consensus among the control engineering community. The main goal of this chapter is to address one of the key issues in ASC: how to synthesize test functionals for stability monitoring/inference.

3.1.2 Adaptive switching control with confidence

Although the fundamental problem of how to systematically ensure (i)-(ii) is an open one, the choice of the test functional plays a crucial role in improving quality of an ASC scheme [KS01].

The main positive feature inherent to all early UASC schemes [ST97, SWPS07] is the fact that they give a precise characterization of the confidence level of a proposed controller: If the switching stops onto one of the candidate controllers − say C_f − the feedback-loop (P/C_f) is either input-output stable or it behaves alike. Such a feature, along with the fact that the plant need not to be linear, makes from the robustness viewpoint, UASC a significant improvement over pre-existing model-based approaches [Mor95, NB97, HLM$^+$01], whose stability is typically only guaranteed if the unknown plant is tightly approximated by at least one candidate model. However, in addition to stability, there are futher aspects in an ASC system which deserve consideration. Given any $P \in \mathscr{P}$, let $S(P) \subseteq \underline{N}$ be the set of all indices $s \in \underline{N}$ such that (P/C_s) is stable, and $S^c(P)$ the complement of $S(P)$ in $\underline{N}$. In connection with (ii), and with specific reference to the HSL (2.9) [2], the ideal test functional is the one ensuring that, for any $P \in \mathscr{P}$, and $\forall\, t \in \mathbb{Z}_+$

$$\exists\, s : \quad V_s(t) < V_i(t) - h, \qquad \forall\, i \in S^c(P) \tag{3.3}$$

Thus, an important quality factor of a test functional is its capability of reducing the chance that (3.3) fail to hold, *viz.* to infer if a candidate controller would make the loop unstable once switched-on in feedback to the plant.

As exemplified below, (3.3) might not hold − and this is typically what happen, even over long initial intervals [BBMT09] − with test functionals of the type defined in Chapter 2, whereby no

[2] Alternative switching policies are considered in Section 4.3

plant model is used.

Example A $-$ Assume that the true (unknown) plant model is as follows

$$(1-2\,d)\,y(t)=u(t-1), \qquad y(-1)=-1,\ u(-1)=1$$

Consider the family $\mathscr{C}=\{C_1=1,\,C_2=1.5,\,C_3=-1.5\}$ of candidate feedback gains, (only C_2 stabilizes the plant). Each feedback gain, once connected in feedback to the plant, produces the control input $u=C_i(r-y)$. Assume $\sigma(0)=1$ (C_1 initially switched-on), and $r(t)=0$, $t\geq 0$. Consider now a functional of the type as in Section Z (for simplicity $\mu:=0$)

$$V_i(t)=\mathfrak{V}_i(z^t,C_i):=\max\left\{\frac{\left\|(\Sigma_{*/i}[v_i])^\tau\right\|}{\|v_i^\tau\|},\ \tau=0,\cdots,t\right\}, \qquad i\in\underline{N} \tag{3.4}$$

where $\Sigma_{*/i}:v_i\to[\,u\ \varepsilon_i\,]'$, $\varepsilon_i:=v_i-y$, denotes the mixed sensitivity of (P/C_i)

$$\Sigma_{*/i}(d):=\frac{1}{1+P(d)C_i(d)}\begin{bmatrix}C_i(d)\\1\end{bmatrix}=\begin{bmatrix}\mathscr{S}_{*/i}(d)\,C_i(d)\\\mathscr{S}_{*/i}(d)\end{bmatrix} \tag{3.5}$$

with $\mathscr{S}_{*/i}$ the sensitivity function of (P/C_i). Notice that, as long as C_1 is kept in the loop, one has $u(t)=1$, $t\geq 0$. Under the same conditions, $y(t)=-1$, $t\geq 0$. Consequently,

$$|\varepsilon_1(t)|=1,\ |\varepsilon_2(t)|=|\varepsilon_3(t)|=2/3$$

as $v_1(t)=0$, $v_2(t)=-1/3$, and $v_3(t)=-5/3$, $t\geq 0$, with the v_i's computed in accordance with (2.7). Further, $|v_3(t)|>|v_2(t)|$, $t\geq 0$. In words, as long as C_1 is kept in the loop, one has $V_2(t)>V_3(t)$. Thus, as soon as switching is activated, C_3 is switched-on. $\square$

In contrast with early *Performance-based* approaches to ASC [Nus83, Mar85, FB86], UASC is less susceptible to poor transient response, as it does not perform pre-routing. However, in order to make computations possible and avoid pre-routing, UASC forces one to replace conceptually rigorous criteria such as $\pi_{*/i}:=\sup_{\|v_i\|\leq 1}\left\|\Sigma_{*/i}[v_i]\right\|$ with their on-line implementable counterparts as in (3.4). The latter, moreover, is not related with real, but merely with virtual operating conditions. In particular, since $C_i[v_i-y]=u$, $i\in\underline{N}$, under the assumption that C_i be CSCI (see Section 2.2), functionals as in (3.4) turn out to be bounded if u is such, irrespective of the stability of (P/C_i).

This situation looks antithetic to *Model-based* ASC [Mor95, NB97, HLM+01, FAP06, KI08], where transients can be made small at the cost of dense nominal model distributions. Let $\mathscr{M}:=\{M_i,i\in\underline{N}\}$ be a finite family of N strictly causal LTI dynamic models M_i approximating $\mathscr{P}$,

$$M_i(d):=B_i(d)/a_i(d), \qquad i\in\underline{N} \tag{3.6}$$

where $B_i(d)$ and $a_i(d)$ are polynomials with strictly Hurwitz g.c.d., $a_i(0)=1$. In multi-model ASC (MMASC, for short) schemes [Mor95, NB97, HLM+01, PJ01, ZMF00, Lib03], the M_i's, along with the associated C_i's, form a finite family

$$\mathscr{R} := \{(M_i/C_i), \ \ i \in \underline{N}\}$$

of *internally* stable feedback-loops, each designed to fulfill desirable prescriptions. Such prescriptions might only be of qualitative type, and, hence, C_i, in general, need not be optimal relatively to any specific performance index. Under appropriate conditions – see [ABH$^+$00] and Section 3.2 – it is possible to set $\mathscr{M}$ dense enough in $\mathscr{P}$ so that – via a *finite* family $\mathscr{R}$ – every admissible process model $P \in \mathscr{P}$ be tightly approximated by at least one nominal model, as well as satisfactorily controlled by at least one of the C_i's.

On that basis, most of current MMASC schemes exploit the idea of certainty equivalence from parameter adaptive control – extended from tuning to switching – [Mor95, NB97, Hes98]: At every time instant, one selects the controller corresponding to the nominal process model whose associated identification error is the smallest. Typically, the identification error is set equal to the *prediction error*

$$\epsilon_i(t) := a_i(d)\, y(t) - B_i(d)\, u(t) = y(t) - \hat{y}_i(t) \tag{3.7}$$

based on the nominal plant model M_i, where $\hat{y}_i(t) := [\, 1 - a_i(d)\,]\, y(t) + a_i(d)\, u(t)$ denotes the *plant output prediction* at time t based on M_i. Then, a conceivable functional on which to base the controller selection is as follows [3]

$$V_i(t) = \mathfrak{V}_i(z^t, M_i) := \frac{1}{t+1} \sum_{k=0}^{t} \epsilon_i^2(k), \qquad i \in \underline{N} \tag{3.8}$$

The motivation for certainty equivalence is self-evident: If the nominal process model associated to the smallest identification error (3.8) – say M_σ – is a good representation of the true process model, the closed-loop behavior (P/C_σ) is likely to be close as well to (M_σ/C_σ). As a matter of fact, the chance that destabilizing controllers be switched-on in feedback to the plant (equivalently, that (3.3) do not hold) can be drastically reduced whenever prior information on the uncertainty set $\mathscr{P}$ is available. The most obvious occurrence of such a phenomenon is the well-known *plant-model matching* condition [Mor96, HLM$^+$01]: If the process model exactly matches one of the N nominal process models – say M_σ – and there are no noises/disturbances, one has $\epsilon_\sigma(t) = 0$, $t \geq \max\{\deg a,\, \deg B\}$.

Merits and shortcomings of MMASC scheme based on test functionals such as (3.8) are briefly exemplified below.

[3] Alternatively, one endows $V_i(t)$ having an exponential forgetting factor, as in (2.22).

Example B $-$ Consider the same problem as in Example A, with

$$(1-2\,d)\,y(t) = u(t-1), \qquad y(-1) = u(-1) = 0$$

and the family $\mathscr{C} = \{C_1 = 1,\, C_2 = 1.5\}$ of candidate feedback gains (only C_2 stabilizes the plant). Again, assume $\sigma(0) = 1$ (C_1 initially switched-on), while $r(t) = 1,\ t \geq 0$. Consider first the case where $a_1(d) := 1 - 1.5\,d$, $a_2(d) := 1 - 2\,d$, and $B_1(d) = B_2(d) := d$ (M_2 exactly matches the process model). It is simple to verify that $\sigma(t) = 2,\ t \geq 2$, *viz.* one rapidly selects C_2 as the final controller. Consider now the case where $a_2(d) := 1 - d$. Notice that, as long as C_1 is kept in the loop, one has

$$y(t) = t,\ u(t) = 1 - t, \qquad t \geq 0$$

Therefore, it is simple to conclude that $\epsilon_1(0) = \epsilon_2(0) = 0$, ϵ_i as in (3.7), while

$$\epsilon_1(t) = 0.5\,(t-1),\ \epsilon_2(t) = t - 1, \qquad t \geq 1$$

Consequently, by adopting a cost-based switching logic, such as the HSL (2.9), C_1 is never switched off. □

In MMASC schemes, at the cost of dense nominal model distributions, transients can be typically made small: a property this, which, in most of the cases, cannot be achieved if no information on P is extracted from z. On the other hand, as exemplified above and pointed out in [MA01, HLM$^+$01, SS08], MMASC schemes fail to yield stability robustness against unmodeled dynamics in-the-large. The major shortcoming of multi-model schemes based on pure process identification manifests itself in total the absence of sinergy between identification and control [Gev93, dHS95, Gev02]: In case of coarse model distributions, at worst, the switched system undergoes instability [SS08]; even if no stability loss occurs, typically, neither convergence to a final controller nor satisfactory schedule are experienced [MA01, AD08].

Next section shows that, by suitably measuring the distance between the candidate loops (P/C_i)'s and the related nominal/reference loops (M_i/C_i)'s, it is possible to construct model-based ASC schemes whereby the positive features of Performance-based control and Model-based control might coexist and their respective deficiencies moderated. It will be shown that the proposed scheme, called *multi-model UASC*, enjoys the following features: as for stability, it preserves the validity assumptions of Theorem 2.2.1, thus allowing application in case of large plant uncertainties; as for performance, it reduces, as in MMASC, both the magnitude and time durations of "learning" transients after start-up in case the plant uncertainty set $\mathscr{P}$ be tightly approximated by $\mathscr{M}$. In addition, as will be better clarified in Chapter 4, multi-model UASC, by embodying a reference-loop identification scheme, is less susceptible than MMASC to poor transients and unsatisfactory schedule even in case of a coarse nominal model distribution.

3.2 Rereference-loop Identification in Multi-model UASC

In the sequel, a manifold of alternative multi-model based UASC schemes will be considered, each one equipped with a different test functional. Their features will be comparatively analyzed mainly in terms of generality of applicability, stability inference and on-line implementability. It will be shown that, among the various conceivable schemes, the one deriving from V_i^{RC} in (3.35) is the only one whereby the positive features of UASC and MMASC can coexist.

For the sake of clarity, in the remainder of this chapter, zero plant initial conditions as well as zero noises/disturbances will be assumed, the general case being deferred to Chapter 4.

Let $\mathscr{R} := \{(M_i/C_i),\ i \in \underline{N}\}$ be a finite family of *internally* stable feedback-loops, each designed to fulfill desirable prescriptions. Hereafter, (M_i/C_i) will be referred to as the i-th "*tuned-loop*" or "*reference-loop*". Given an unknown plant $P \in \mathscr{P}$, one of the main steps in a multi-model UASC is to carry out a *reference-loop identification* task, *viz.*, select a candidate controller C_σ in such a way that (P/C_σ) behave as closest as possible to one of the candidate reference-loops in $\mathscr{R}$. Hence, roughly speaking, prescriptions (i)-(ii) will be approached as follows: Given an uncertain plant $P \in \mathscr{P}$, select the index $\sigma \in \underline{N}$ such that:

(iii) The behavioral data produced by (P/C_σ) in response to the virtual reference v_σ are close to the ones produced by (M_σ/C_σ) in accordance to the reference-loop identification criterion

$$\sigma := \arg\min_{i\in\underline{N}} \sup_{v_i\neq 0} \frac{\|(P/C_i)_0[v_i] - (M_i/C_i)_0[v_i]\|}{\|(F_i)_0[v_i]\|} \tag{3.9}$$

where, by the sake of simplicity no time-argument is shown, and F_i denotes an LTI filter such that $v_i \neq 0$ implies $\|(F_i)_0[v_i]\| > 0$ [4].

[4] As in UASC, the normalized form (3.9) appears as a natural choice in view of the fact that functionals deriving from (3.9) make it possible to assess the suitability of a candidate controller C_i, whenever relevant behavioral features of (P/C_i) depend on feedback system induced gains. As will be shown soon, functionals related to (3.9) are indeed of the form

$$V_i(t) := \max\left\{ \frac{\|((\Delta_{*/i})_0[\xi_i])^\tau\|}{\|\xi_i^\tau\|},\ \tau = 0, \cdots, t \right\}, \qquad i \in \underline{N} \tag{3.10}$$

where the vector-valued variables $\eta_i(t) := (\Delta_{*/i})_0[\xi_i](t)$ and $\xi_i(t)$ are computable from the data sequence z^t. The understanding, here, is that the upper-bound to (3.10) given by the induced gain $\|\Delta_{*/i}\|_{\mathrm{ind}}$ of the transformation $\Delta_{*/i}$ from $\xi_i \in \mathcal{N}(\mathbb{Z}_+)$ to $\eta_i \in \mathcal{N}(\mathbb{Z}_+)$, provides a measure of the distance between (P/C_i) and the i-th tuned-loop (M_i/C_i). This implies that it be possible to assess the suitability of C_i as a feedback controller of P from the magnitude of $V_i(t)$ which gives an estimate from below of $\|\Delta_{*/i}\|_{\mathrm{ind}}$.

Remark 3.2.1 – In (3.9), the notation $(P/C_i)_0[v_i]$ is adopted in accordance with the zero noises/disturbances constraint and (2.8), which yields $(P/C_{\sigma(\cdot)})[w] = (P/C_i)[w_i]$ where $w = \{r, n_u, n_y\}$ and $w_i = \{v_i, n_u, n_y\}$.

Remark 3.2.2 – In accordance with the mentioned restriction to zero initial conditions for (P/C_i) – imposed on (M_i/C_i) – for simplicity of notation, unless otherwise needed to avoid possible ambiguities, we will drop the subscript "0" for all the operators in (3.9) in the remainder of this chapter.

In order to ensure constant disturbance rejection and zero-offset (controllers equipped with an "integral action" [Mos95]), the quantities in (3.9) are defined as follows.

$$(M_i/C_i)[v_i](t) := [\,\delta u_{i/i}(t)\ y_{i/i}(t)\,]' =: z_{i/i}(t)$$

which means that the N nominal models in (3.6), and the related C_i's in (2.5) are represented in incremental form as

$$M_i: \quad A_i(d)y_{i/i}(t) = B_i(d)\delta u_{i/i}(t), \quad A_i(d) := a_i(d)\,(1-d) \tag{3.11}$$

$$C_i: \quad R_i(d)\delta u_{i/i} = S_i(d)(v_i(t) - y_{i/i}(t)), \quad R_i(d) := r_i(d)\,/(1-d) \tag{3.12}$$

with $B_i(d)$ and $A_i(d)$ polynomials with strictly Hurwitz g.c.d.. Likewise,

$$(P/C_i)[v_i](t) = [\,\delta u(t)\ y(t)\,]' =: z(t)$$

which amounts to adopting for the plant P in (3.2) the incremental form

$$P: \ A(d,\theta)\,y(t) = B(d,\theta)\,\delta u(t) + \delta w(t), \quad A(d,\theta) := a(d,\theta)\,(1-d) \tag{3.13}$$

where $B(d,\theta)$ and $A(d,\theta)$, $\theta \in \Theta$, are assumed to have strictly Hurwitz g.c.d. [5]; further, $\delta w(t) := B(d,\theta)\,\delta n_u(t) + a(d,\theta)\,\delta n_y(t)$; the controller C_i operating in $(P/C_i)[v_i]$ is defined as in (3.12) with $z_{i/i}$ changed into z.

Therefore, in view of (3.9), the form taken by $V_i(t)$ in (3.10) is as follows

$$V_i(t) := \max\left\{ \frac{\|\,\tilde{z}_i^\tau\,\|}{\|\,(F_i[v_i])^\tau\,\|},\ \ \tau = 0, \cdots, t \right\}, \qquad i \in \underline{N} \tag{3.14}$$

where $\tilde{z}_i(t) := z(t) - z_{i/i}(t)$, $t \in \mathbb{Z}_+$, and, by definition, $V_i(t)$ is set equal to zero, whenever $\|z^t\| = 0$.

[5] Note that, by Bezout identity [Mos95], $A(d)$ and $B(d)$ with strictly Hurwitz g.c.d. $D(d)$ amount to the existence of polynomials $X(d)$ and $Y(d)$ such that $X\,A + Y\,B = D$. This implies that $X\,a\,\delta u + Y\,a\,y = D\,u$, where $A(d) = a(d)(1-d)$. Consequently, $\mathcal{N}$-stability relatively to $z := \{\,\delta u,\ y\,\}$ implies $\mathcal{N}$-stability relatively to $\overline{z} := \{\,u,\ y\,\}$, *i.e.*, (2.3) of Chapter 2. As a matter of fact, similar conclusions can be obtained relatively to (3.2).

Table 3.1: Reference-loop *identification functionals* V_i's and related F_i-choices

$V_i^{\mathrm{PA}}(t)$ in (3.17)	$F_i[v_i](t) = [\, \delta u(t) \;\; y(t) - v_i(t) \,]' =: \eta_{*/i}(t)$
	The filter F_i yields what is referred to as the PA-*percentage discrepancy* relatively to the *potential* loop performance data $\eta_{*/i}$. Its related test functional V_i^{PA} is an estimate from below of the induced gain of the operator (3.15). The adopted terminology stems from the fact that F_i involves the potential **(P)** loop (P/C_i) and the operator (3.15) stems from considering robustness against the additive **(A)** uncertainty on P relatively to M_i, the latter considered as the plant nominal model;
$V_i^{\mathrm{PC}}(t)$ in (3.23)	$F_i[v_i](t) = [\, \delta u(t) \;\; y(t) \,]' =: z(t)$
	The filter F_i yields what is referred to as the PC-*percentage discrepancy* relatively to the *potential* loop data z. Its related test functional V_i^{PC} is an estimate from below of the induced gain of the operator (3.22). Here, the normalization factor involves the potential **(P)** loop (P/C_i), while the operator (3.22) stems from considering robustness against the coprime **(C)** factor uncertainty on P relatively to M_i;
$V_i^{\mathrm{RA}}(t)$ in (3.29)	$F_i[v_i](t) = [\, \delta u_{i/i}(t) \;\; y_{i/i}(t) - v_i(t) \,]' =: \eta_{i/i}(t)$
	The filter F_i yields what is referred to as the RA-*percentage discrepancy* relatively to the *reference* loop data $\eta_{i/i}$. Its related test functional V_i^{RA} is an estimate from below of the induced gain of the operator (3.28). In such a case, the normalization factor involves the reference **(R)** loop (M_i/C_i), while the operator (3.28) stems from considering robustness against the additive **(A)** uncertainty on P relatively to M_i;
$V_i^{\mathrm{RC}}(t)$ in (3.35)	$F_i[v_i](t) = [\, \delta u_{i/i}(t) \;\; y_{i/i}(t) \,]' =: z_{i/i}(t)$
	The filter F_i yields what is referred to as the PC-*percentage discrepancy* relatively to the *reference* loop data $z_{i/i}$. Its related test functional V_i^{RC} is an estimate from below of the induced gain of the operator (3.34). Here, the normalization factor involves the reference **(R)** loop (M_i/C_i), while the operator (3.34) stems from considering robustness against the coprime **(C)** factor uncertainty on P relatively to M_i.

In the sequel, $V_i(t)$ in (3.14) will be referred to as the the i-th "*test functional*" or "*identification functional*". Natural functionals $V_i(t)$ for reference-loop identification of Fig. 3.1 are the

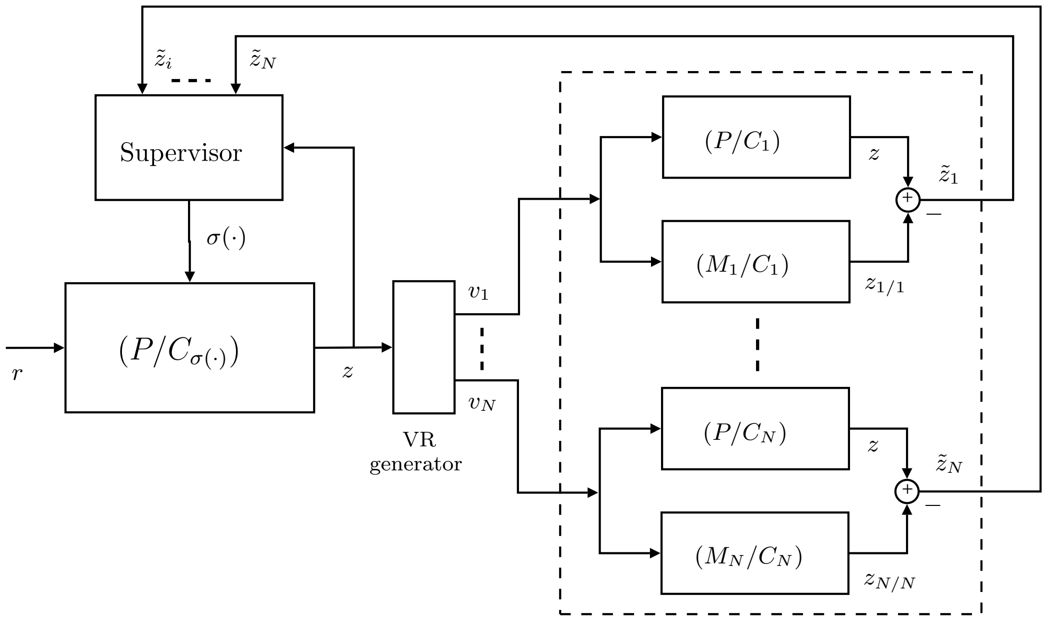


Figure 3.1: Block diagram of UASC for reference-loop identification.

ones characterized by the F_i-choices of Table 3.1.

In the sequel, both functionals V_i^{PA} and V_i^{PC} whose denominators involve the i-th potential loop, will be referred to as P-*percentage discrepancies*, while V_i^{RA} and V_i^{RC}, with normalizations involving the i-th reference-loop will be denoted as R-*percentage discrepancies*. In this respect, the main difference between P-percentage and R-percentage discrepancies is as follows. The former ones only test necessary conditions for instability of the potential loop (P/C_i) in that, given a candidate C_i, if (P/C_i) is unstable, V_i, deriving from a P-percentage discrepancy, necessarily exceeds a known threshold for some $r \in \mathscr{N}(\mathbb{Z}_+)$ and $\sigma \in \mathfrak{S}$. Consequently, with P-percentage discrepancies, the family $\mathscr{R}$ of reference-loops must be dense enough in $\mathscr{P}$ in order to ensure that, for any $P \in \mathscr{P}$, there be at least one reference-loop (M_i/C_i) such that the related test functional V_i be smaller than or equal to such a threshold, for every $r \in \mathscr{N}(\mathbb{Z}_+)$ and $\sigma \in \mathfrak{S}$. On the contrary, R-percentage discrepancies test necessary and sufficient stability conditions in that, given a candidate C_i, (P/C_i) is stable if and only if V_i, deriving from an R-percentage discrepancy, takes on finite values, for every $r \in \mathscr{N}(\mathbb{Z}_+)$ and $\sigma \in \mathfrak{S}$. Hence, $\mathscr{R}$ must only ensure that for any $P \in \mathscr{P}$ there be at least one candidate controller C_i whose related test functional V_i remain bounded, for every $r \in \mathscr{N}(\mathbb{Z}_+)$ and $\sigma \in \mathfrak{S}$.

3.2.1 P-percentage discrepancies

Given the reference-loop (M_i/C_i), the potential loop (P/C_i) can be conceived as a perturbation of (M_i/C_i), the perturbation being caused by the possible difference between P and M_i. By P-percentage discrepancies, it is possible to test necessary conditions for instability of a potential loop (P/C_i) based on small-gain stability arguments: the specific F_i-choice in (3.14) (yielding either z or $\eta_{*/i}$) depends on the adopted description of the plant uncertainty.

Additive uncertainty

Let the uncertain plant P be represented in terms of an *additive perturbation* $P - M_i$ of the nominal model M_i. A result of small-gain stability [DDB95], is that a sufficient condition for $\mathcal{N}$-stability of (P/C_i) is that $\|\Xi_{*/i}\|_{\text{ind}} < 1$, where

$$\Xi_{*/i}(d) := \frac{(\,P(d) - M_i(d)\,)C_i(d)}{1 + M_i(d)\,C_i(d)} = \frac{(\,B(d)A_i(d) - B_i(d)A(d)\,)\,S_i(d)}{A(d)\,\chi_{i/i}(d)} \tag{3.15}$$

with $\chi_{i/i} := A_i\,R_i + B_i\,S_i$, the characteristic polynomial of (M_i/C_i). It can be seen that, under the zero initial condition constraint, one has

$$\tilde{z}_i(t) = \tilde{z}_{i0}(t) = \left(\text{diag}\left\{\Xi_{*/i}\right\}\right)_0\,[\eta_{*/i}](t) \tag{3.16}$$

Hence, in order to get an estimate from below of $\|\Xi_{*/i}\|_{\text{ind}}$, one can set $F_i[v_i] := \eta_{*/i}$, yielding

$$V_i^{\text{PA}}(t) := \max\left\{\frac{\|\tilde{z}_{i0}^\tau\|}{\left\|\eta_{*/i}^\tau\right\|},\ \tau = 0, \cdots, t\right\}, \qquad i \in \underline{N} \tag{3.17}$$

In view of (3.15), in order to be able to deduce $\mathcal{N}$-stability of (P/C_i) from the magnitude of V_i^{PA}, the polynomial $A_i(d)$ must cancel all the (possible) unstable roots of $A(d)$. Consequently, the following assumption is needed.

E1. The denominator $A(d,\theta)$ of $P \in \mathscr{P}$ satisfies

$$A(d,\theta) = A^+(d)\,A^-(d,\theta)$$

with $A^+(d)$ fixed and known, and $A^-(d,\theta)$ strictly Hurwitz.

Under E1, it is possible to select each nominal model M_i in such a way that $A_i(d) := A^+(d)\,A_i^-(d)$, with $A_i^-(d)$ strictly Hurwitz. Hence, in accordance with Fig. 3.2, $M_i(d) = M_i^-(d)/A^+(d)$, and

$$P(d) = \frac{1}{A^+(d)}\left(M_i^-(d) + \Delta_i(d)\right), \quad M_i^-(d) := B_i(d)/A_i^-(d) \tag{3.18}$$

Therefore, if each $A_i(d)$ is selected in such a manner, $\Xi_{*/i}(d)$ turns out to be

$$\Xi_{*/i}(d) := A_i^-(d)\,S_i(d)\,\chi_{i/i}^{-1}(d)\,\Delta_i(d) =: H_{i/i}(d)\,\Delta_i(d) \tag{3.19}$$

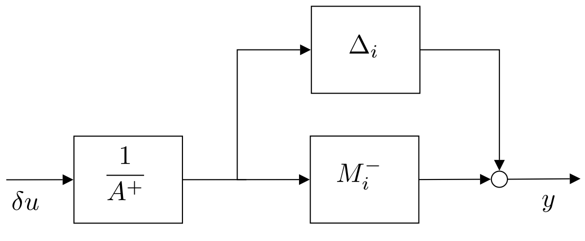


Figure 3.2: Additive model error representation.

Thus, under E1, V_i^{PA} can be used as an estimate from below of $\|\Xi_{*/i}\|_{\mathrm{ind}}$. Since $\|\Xi_{*/i}\|_{\mathrm{ind}} \geq 1$ is a necessary condition for instability of (P/C_i), $\mathscr{M}$ must be set dense enough in $\mathscr{P}$, so as to ensure that, for any $P \in \mathscr{P}$, there exist indices $i \in \underline{N}$, yielding stable loops (P/C_i) such that $\left\|\Xi_{*/i}\right\|_{\mathrm{ind}} \leq \beta < 1$.

Proposition 3.2.1 – Let Θ be a compact set, and $\theta \to P(\theta)$ continuous on Θ. Then, under condition E1, for any positive real β, there always exists a finite model distribution $\mathscr{M}$ such that

$$\max_{P \in \mathscr{P}} \min_{i \in S(\mathscr{P})} \left\|\Xi_{*/i}\right\|_{\mathrm{ind}} =: \beta_{\mathrm{PA}} \leq \beta \tag{3.20}$$

Proof. Notice that the induced norm on $\mathcal{N}(\mathbb{Z}_+)$ of a finite-dimensional LTI stable operator (*i.e.* with absolutely summable pulse response) can be upper-bounded by its $\mathcal{H}_\infty$ norm [DDB95], which depends continuously on θ. Thus, under continuity assumption of the map $\theta \to P(\theta)$, for every $\theta \in \Theta$ and $\beta \in \mathbb{R}_+$, there exists an open ball $\mathfrak{B}(\theta)$ around θ such that $\left\|\Xi_{*/i}\right\|_{\mathrm{ind}} \leq \beta$ (*e.g.* see [ABH+00]). The existence of a finite covering follows from compactness of Θ. $\square$

A model distribution for which such a property holds, will be referred to as a β_{PA}-*dense model distribution* and denoted by $\mathscr{M}(\beta_{\mathrm{PA}})$. Further, assuming $\mathscr{M} = \mathscr{M}(\beta_{\mathrm{PA}})$ and $P \in \mathscr{P}$ given, a specific potential loop (P/C_i) is said to have the β_{PA}-*property* if $\left\|\Xi_{*/i}\right\|_{\mathrm{ind}} \leq \beta_{\mathrm{PA}}$.

Coprime factor uncertainty

An analysis similar to the one considered for the additive uncertainty (3.18) can be carried out in case P be represented in terms of a *coprime factor uncertainty*

$$(A_i(d) - \tilde{A}_i(d))\, y(t) = (B_i(d) + \tilde{B}_i(d))\, \delta u(t)$$

which yields

$$A_i(d)y(t) = B_i(d)\delta u(t) + e_i(t) \tag{3.21}$$

where e_i is the equation error given by $e_i(t) = \tilde{A}_i(d)y(t) + \tilde{B}_i(d)\delta u(t)$ (Fig. 3.3). Let now C_i be a controller which makes (M_i/C_i) internally stable. As beforehand, one uses a result of small-gain

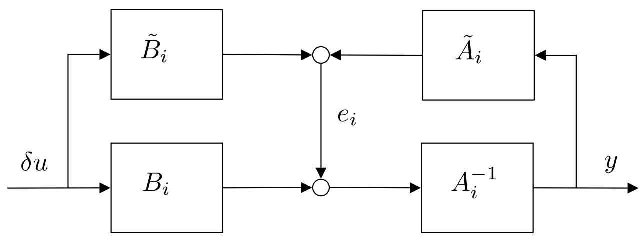


Figure 3.3: Coprime factors model error representation.

stability. Given an uncertainty $\tilde{L}_i(d) := \left[\tilde{B}_i(d)\ \tilde{A}_i(d)\right]$ on the plant coprime factors, a sufficient condition for $\mathcal{N}$-stability of (P/C_i) is that $\|\Psi_{*/i}\|_{\text{ind}} < 1$, where

$$\Psi_{*/i}(d) := \frac{[-S_i(d) \quad R_i(d)]'}{\chi_{i/i}(d)} \left[\tilde{B}_i(d) \quad \tilde{A}_i(d)\right] =: Q_{i/i}(d)\tilde{L}_i(d) \tag{3.22}$$

Now, it can be seen that $\tilde{z}_i = \tilde{z}_{i0} = \left(\Psi_{*/i}\right)_0 [z]$. Thus, the problem of inferring stability of a candidate loop (P/C_i) can be approached by setting in (3.14) $F_i[v_i] := z$, *viz.*

$$V_i^{\text{PC}}(t) := \max\left\{\frac{\|\tilde{z}_{i0}^{\tau}\|}{\|z^{\tau}\|},\ \tau = 0, \cdots, t\right\}, \qquad i \in \underline{N} \tag{3.23}$$

Since $\left\|\Psi_{*/i}\right\|_{\text{ind}} \geq 1$ is a necessary condition for instability of (P/C_i), $\mathscr{M}$ must be set dense enough in $\mathscr{P}$, so as to ensure that, for any $P \in \mathscr{P}$, there are indices $i \in \underline{N}$ yielding stable loops (P/C_i) with $\left\|\Psi_{*/i}\right\|_{\text{ind}} \leq \beta < 1$. In accordance with the next proposition, (3.23) has the advantage over (3.17) of handling also *unknown* plant unstable perturbations.

Proposition 3.2.2 – Let Θ be a compact set, and $\theta \to P(\theta)$ continuous on Θ. Then, for any positive real β, there always exists a finite model distribution $\mathscr{M}$ such that

$$\max_{P \in \mathscr{P}} \min_{i \in S(\mathscr{P})} \left\|\Psi_{*/i}\right\|_{\text{ind}} =: \beta_{\text{PC}} \leq \beta \tag{3.24}$$

□

According to Propositions 3.2.1 and 3.2.2, boundedness of (3.17) and (3.23) holds irrespective of the stability of (P/C_i). Thus, in order to ensure, via either (3.17) or (3.23), falsification of destabilizing controllers, one needs to set the model distribution dense enough in $\mathscr{P}$, *viz.* β_{PA} (or β_{PC}) sufficiently small.

Next theorem captures the basic properties ensured by the P-percentage functionals.

Theorem 3.2.1 – *Let $z = \{\delta u, y\}$ be the vector-valued sequence of I/O plant data and σ the switching sequence resulting from (2.6) and the HSL (2.9), with P as in (3.13) – under zero initial conditions and zero noises/disturbances – and test functionals (3.14). Then, under a model distribution $\mathscr{M}(\beta_{PA})$, $\beta_{PA} < 1 - h$, β_{PA} as in (3.20) [$\mathscr{M}(\beta_{PC})$, $\beta_{PC} < 1 - h$, β_{PC} as in (3.24)], the switching stops in finite time and $(P/C_{\sigma(\cdot)})$ is $\mathcal{N}$-stable. Further,*

i) The total number of switches N_σ is bounded as follows

$$N_\sigma \leq N \lceil \beta_{PA}/h \rceil \quad [N_\sigma \leq N \lceil \beta_{PC}/h \rceil] \tag{3.25}$$

where $\lceil \alpha \rceil$ denotes the smallest integer greater than or equal to $\alpha \in \mathbb{R}_+$;

ii) The occurrence of the condition, $i \in S^c(P)$,

$$V_i^{PA}(t) > \beta_{PA} \quad [V_i^{PC}(t) > \beta_{PC}] \tag{3.26}$$

ensures that no destabilizing controller is switched-on at and after time t;

iii) The occurrence of the condition, $i \in S^c(P)$,

$$V_i^{PA}(t) > \beta_{PA} + h \quad [V_i^{PC}(t) > \beta_{PC} + h] \tag{3.27}$$

ensures that $\sigma(\tau) \in S(P)$, $\tau \geq t$, and the offset-free property to $(P/C_{\sigma(\cdot)})$;

iv) If $h > \beta_{PA}$ [$h > \beta_{PC}$], each candidate controller can be switched-on no more than once, and any stabilizing controller with the β_{PA} [β_{PC}]-property, once switched-on in feedback with the plant, will be never switched-off thereafter. □

To maintain continuity, we defer the proof of Theorem 3.2.1 to Appendix A.

3.2.2 R-percentage discrepancies

A natural and effective way to infer controller suitability from data z consists of endowing a test functional V_i having the property that (P/C_i) is $\mathcal{N}$-stable if and only if V_i takes on finite values, for every $r \in \mathcal{N}(\mathbb{Z}_+)$ and $\sigma \in \mathfrak{S}$. Section 3.2.1 showed that this is not the case for P-percentage discrepancies. As shown next, the above desired property for V_i holds for R-percentage discrepancies. The specific F_i-choice in (3.14) (here either $z_{i/i}$ or $\eta_{i/i}$) depends on the adopted description of the plant uncertainty.

Additive uncertainty

Let the uncertain plant P be represented in terms of the nominal model M_i and its related *additive uncertainty.* In Section 3.2.1 it was shown that, under condition E1 (see comments before

(3.15)), it is possible to select $\mathscr{M}$ in such a way that, by letting $F_i\,[v_i] := \eta_{*/i}$, the functional V_i^{PA} provides a margin for stability of the potential loop (P/C_i). Now, under E1 and the same choice for the M_i's as in (3.18), for the map from $\eta_{i/i}$ to $\tilde{z}_i$, one finds $\tilde{z}_i(t) = -\mathrm{diag}\left\{\Lambda_{*/i}\right\}[\eta_{i/i}](t)$, where

$$\Lambda_{*/i}(d) := A^{-}(d)\, S_i(d)\, \chi_{*/i}^{-1}(d)\, \Delta_i(d) =: H_{*/i}(d)\, \Delta_i(d) \tag{3.28}$$

with $\chi_{*/i} := A\,R_i + B\,S_i$, the characteristic polynomial of (P/C_i). Accordingly, by letting $F_i\,[v_i] := \eta_{i/i} = \eta_{*/i} - \tilde{z}_{i0}$, one has $\tilde{z}_i = \tilde{z}_{i0} = -\left(\mathrm{diag}\left\{\Lambda_{*/i}\right\}\right)_0\left[\eta_{i/i}\right]$. Hence,

$$V_i^{\mathrm{RA}}(t) := \max\left\{ \frac{\|\tilde{z}_{i0}^{\tau}\|}{\left\|(\eta_{*/i} - \tilde{z}_{i0})^{\tau}\right\|},\ \ \tau = 0, \cdots, t \right\}, \qquad i \in \underline{N} \tag{3.29}$$

In contrast with V_i^{PA}, boundedness of V_i^{RA} depends on Hurwitzianity of $\chi_{*/i}$, *viz.* a potential candidate loop (P/C_i) is stable if and only if V_i^{RA} takes on finite values, for every $r \in \mathcal{N}(\mathbb{Z}_+)$ and $\sigma \in \mathfrak{S}$.

Proposition 3.2.3 – Let Θ be a compact set, and $\theta \to P(\theta)$ continuous on Θ. Then, under condition E1, for any positive real β, there always exists a finite model family $\mathscr{M}$ such that

$$\max_{P\in\mathscr{P}}\ \min_{i\in S(\mathscr{P})} \left\|\Lambda_{*/i}\right\|_{\mathrm{ind}} =: \beta_{\mathrm{RA}} \leq \beta \tag{3.30}$$

□

Remark 3.2.3 – An interesting interpretation for V_i^{RA} in (3.29) is in order. Let

$$\pi_{*/i}(t) := \|((\Sigma_{*/i})_0\,[v_i])^t\| \,/\, \|v_i^t\|, \quad \pi_{i/i}(t) := \|((\Sigma_{i/i})_0\,[v_i])^t\| \,/\, \|v_i^t\|$$

where $\Sigma_{*/i}$ and $\Sigma_{i/i}$ denote the mixed-sensitivity matrices of (P/C_i) as in (3.5), and, respectively, (M_i/K_i), *viz.* (3.5) with P changed into M_i. As can be checked,

$$|\pi_{*/i}(t) - \pi_{i/i}(t)| \leq V_i^{\mathrm{RA}}(t)\, \pi_{i/i}(t), \qquad t \in \mathbb{Z}_+ \tag{3.31}$$

In view of (3.31), V_i^{RA} can be interpreted as an upper-bound on the percentage performance *loss*, caused by replacing the i-th nominal model M_i with the plant P. In particular, at the synthesis stage, $\mathscr{M}(\beta_{\mathrm{RA}})$ can be set dense enough so as to ensure that, for any $P \in \mathscr{P}$,

- $\exists\, i \in \underline{N}$: $\quad |\pi_{*/i}(t) - \pi_{i/i}(t)| \ll \pi_{i/i}(t)$, for every $t \in \mathbb{Z}_+$, where $\pi_{i/i}$ is such that
- $|\pi_{i/i}(t) - \pi_i^{\circ}(t)| \ll \pi_i^{\circ}(t), \quad t \in \mathbb{Z}_+$
  where $\pi_i^{\circ} := \pi_{M_i/C_i^{\circ}}$ is the performance associated to $C_i^{\circ} := \arg\min_C \pi_{M_i/C}$;

In words, the second prescription implies that the loss of performance of (M_i/K_i) is relatively small with respect to π_i°, while the first one requires that the same property hold between real and designed performance [6]. One might expect that a change of the denominator of (3.29) into $\|v_i^t\|$ would provide a convenient simplification. However, as the reader may check, under such a choice

$$V_i(t) := \max \left\{ \frac{\| \left(\left(\text{diag } \{\Lambda_{*/i}\} [\Sigma_{i,i}]\right)_0 [v_i]\right)^\tau \|}{\|v_i^\tau\|}, \ \tau = 0, \cdots, t \right\}, \quad i \in \underline{N} \tag{3.32}$$

Consequently, one sees that the dependence of (3.32) on $\Sigma_{i/i}$, makes $V_i(t)$ in (3.14) affected by the magnitude of the latter. In turns, such a magnitude might be quite different at the various i's, hence, introduce undesirable biases in the controller switching mechanism, and also reduce performance, since (3.32) only implies

$$|\pi_{*/i}(t) - \pi_{i/i}(t)| \leq V_i(t), \qquad t \in \mathbb{Z}_+ \tag{3.33}$$

viz. a bound on the performance loss, which is unaffected by the order of the nominal performance.

Coprime factor uncertainty

A convenient way for dealing with possibly unknown unstable modes of the plant, and getting rid of E1 is to replace the denominator of (3.29) with $F_i[v_i] := z_{i/i}$. In fact, let P be represented in terms of a coprime factor uncertainty. Following similar lines as in Section 3.2.1, one can verify that, under the zero initial condition constraint, the transfer matrix from $z_{i/i}$ to $\tilde{z}_{i0}$ coincides with

$$\Omega_{*/i}(d) := \frac{[-S_i(d) \quad R_i(d)]'}{\chi_{*/i}(d)} \begin{bmatrix} \tilde{B}_i(d) \ \tilde{A}_i(d) \end{bmatrix} =: Q_{*/i} \, \tilde{L}_i \tag{3.34}$$

where $\tilde{L}_i = [\, \tilde{B}_i \ \tilde{A}_i \,]$ denotes a stable uncertainty on plant model coprime factors. Thus, $F_i[v_i] := z_{i/i}$ yields $\tilde{z}_i = \tilde{z}_{i0} = (\Omega_{*/i})_0[z_{i/i}]$, with $z_{i/i} = z - \tilde{z}_{i0}$. Accordingly, one can set

$$V_i^{\text{RC}}(t) := \max \left\{ \frac{\|\tilde{z}_{i0}^\tau\|}{\|(z - \tilde{z}_{i0})^\tau\|}, \ \tau = 0, \cdots, t \right\}, \qquad i \in \underline{N} \tag{3.35}$$

Similarly to V_i^{RA}, but in contrast with V_i^{PC}, boundedness of V_i^{RC} depends on Hurwitzianity of $\chi_{*/i}$, As a result of next proposition, any finite β_{RC} (instead of $\beta_{\text{PC}} < 1$) implies that for any $P \in \mathscr{P}$ there exist indices $i \in \underline{N}$ yielding stable loops (P/C_i)'s.

Proposition 3.2.4 – Let Θ be a compact set, and $\theta \to P(\theta)$ continuous on Θ. Then, for any positive real β, there always exists a finite model family $\mathscr{M}$ such that

$$\max_{P \in \mathscr{P}} \min_{i \in S(\mathscr{P})} \left\|\Omega_{*/i}\right\|_{\text{ind}} =: \beta_{\text{RC}} \leq \beta \tag{3.36}$$

□

[6] It is worth pointing out that in [dHS95] a similar analsys is carried out w.r.t. $\pi_{*/i}(t) := E[\,(y(t) - r(r))^2 + u^2(t)\,]$ (E denoting ensemble-average), where $\pi_{*/i}$, $\pi_{i/i}$ and $|\pi_{*/i} - \pi_{i/i}|$ are referred to as the *achieved performance*, the *designed performance*, and, respectively, the *performance degradation*.

Note that, the identification functional V_i^{RC} in (3.35) also provides a direct stability measure of the candidate loop (P/C_i), since $V_i^{\mathrm{RC}}(t) \le \|T_{*/i}\|_{\mathrm{ind}}$, $t \in \mathbb{Z}_+$, where $T_{*/i}(d)$ coincides with the generalized sensitivity matrix of (P/C_i)

$$T_{*/i}(d) := (1 + P\,C_i)^{-1} \begin{bmatrix} -PC_i & C_i \\ P & -1 \end{bmatrix} \tag{3.37}$$

Next theorem captures the basic properties ensured by the R-percentage functionals.

Theorem 3.2.2 – *Let $z = \{\delta u, y\}$ be the vector-valued sequence of I/O plant data and σ the switching sequence resulting from (2.6) and the HSL (2.9), with P as in (3.13) – under zero initial conditions and zero noises/disturbances – and test functionals (3.14). Then, under a model distribution $\mathscr{M}(\beta_{RA})$, β_{RA} as in (3.30) [$\mathscr{M}(\beta_{RC})$, β_{RC} as in (3.36)], the switching stops in finite time and $(P/C_{\sigma(\cdot)})$ is $\mathcal{N}$-stable. Furthermore, properties i) - iv) of Theorem 3.2.1 hold true with reference to β_{RA} [β_{RC}].*

Proof. Because initial conditions for all the operators in (3.9) are zero, it follows that $z^t = 0$ implies $\tilde{z}_{i0}^t = z_{i/i}^t = 0$. Further, recalling that $A(0) = A_i(0) = 1$ and $B(0) = B_i(0) = 0$, $\tilde{L}_i(0) = 0$. Thus, at the first time instant t where $z(t) \neq 0$, one has $\tilde{z}_{i0}(t) = 0$ and $z_{i/i}(t) = z(t)$. As a consequence, one can conclude that the test functionals V_i^{RA} and V_i^{RC} always take on finite values, their denominators always being greater than zero whenever $\|z^t\| > 0$. The rest of the proof can be given along the same lines as in Theorem 3.2.1, and therefore omitted. □

3.3 Comparative Assessment of Test Functionals

Table 3.2 depicts final comparative features of the test functionals considered in this chapter. In this respect, V_i^S is used for denoting the original model-free test functional à la Safonov, as in (3.4) and Chapter 2,

$$V_i^S(t) := \max \left\{ \frac{\left\|((\Sigma_{*/i})_0[v_i])^\tau\right\|}{\|v_i^\tau\|},\ \tau = 0, \cdots, t \right\}, \qquad i \in \underline{N} \tag{3.38}$$

As elaborated hereafter, there are various reasons for assessing V_i^{RC} as the most effective functional among the possible alternatives.

1) The reason for preferring V_i^{RC} over V_i^{PA} and V_i^{RA} is twofold. First, V_i^{RC} allows safe operation under all possible circumstances provided that solely Problem Feasibility hold. In particular, adoption of V_i^{RC} does not require that possible plant unstable modes be known. This is in contrast with the use of V_i^{PA} or V_i^{RA}, which ask for the prior knowledge of the plant unstable modes. Additionally, V_i^{RC} does not need an explicit computation of the v_i's. In fact, as will be better clarified in

Table 3.2: Comparison among functionals V_i's

Functionals	VR computation	Uncertainty	N-S conditions $^{(*)}$ for $\mathcal{N}$-stability	N-S conditions $^{(*)}$ for the existence of $\mathscr{M}(\beta)$
V_i^S	Necessary	Non model-based	PF	Non model-based
V_i^{PA}	Necessary	Additive	$\beta_{\mathrm{PA}} < 1-h \;\; (\Rightarrow \mathrm{PF})$	E1
V_i^{PC}	Unnecessary	Coprime factors	$\beta_{\mathrm{PC}} < 1-h \;\; (\Rightarrow \mathrm{PF})$	None
V_i^{RA}	Necessary	Additive	$\beta_{\mathrm{RA}} < \infty \;\; (\Rightarrow \mathrm{PF})$	E1
V_i^{RC}	Unnecessary	Coprime factors	PF	None

$^{(*)}$: Assuming compactness of Θ and continuity of the map $\theta \to \Pi(\theta)$
N-S $\equiv$ Necessary and Sufficient; PF $\equiv$ Problem Feasibility

the next section, computation of V_i^{RC} only hinges upon the mere knowledge of z. Consequently, V_i^{RC} is on-line computable even when, as in the case of non-stably invertible controllers, v_i is not well-defined. On the contrary, the latter is required if V_i^{PA} or V_i^{RA} is used.

2) The second comparison concerns V_i^{RC} and V_i^{PC}. Notice that $\Psi_{*/i}$ in (3.22) and $\Omega_{*/i}$ in (3.34) satisfy

$$\Omega_{*/i}(d) - \Psi_{*/i}(d) = \Omega_{*/i}(d)\, \Psi_{*/i}(d)$$

Consequently, the following implications hold:

$$\begin{aligned} \|\Psi_{*/i}\|_{\mathrm{ind}} < \beta_{\mathrm{PC}} \qquad &\Rightarrow \qquad \|\Omega_{*/i}\|_{\mathrm{ind}} < \beta_{\mathrm{PC}}\,(1-\beta_{\mathrm{PC}})^{-1} \\ \|\Omega_{*/i}\|_{\mathrm{ind}} < \beta_{\mathrm{RC}} \qquad &\Rightarrow \qquad \|\Psi_{*/i}\|_{\mathrm{ind}} < \beta_{\mathrm{RC}}\,(1-\beta_{\mathrm{RC}})^{-1} \end{aligned} \tag{3.39}$$

Eq. (3.39) states that, in principle, any switching performance achievable via V_i^{RC} can be also obtained via V_i^{PC} and *vice versa*. Nonetheless, there are other reasons for considering V_i^{RC} preferable to V_i^{PC}. Safe operation with V_i^{PC} requires the condition $\beta_{\mathrm{PC}} < 1-h$. As will be better clarified in next section, satisfaction of such a condition turns out to be a quite problematic task whenever the feedback loop is affected by unknown noises/disturbances. In contrast with it, safe operation with V_i^{RC} is solely subject to Problem Feasibility.

3) Despite the fact that asymptotic properties related to the use of either functionals rely on the same condition, *viz.* Problem Feasibility, there are two main reasons for preferring V_i^{RC} to V_i^S. First, as already pointed out, V_i^{RC} can be used even when v_i is not well-defined, a situation the latter under which the adoption of V_i^S forces the designer to adopt variants [MCMS07] of the basic scheme that might reduce performance [BBMT09]. Second, in accordance with Theorem 3.2.2, V_i^{RC} can be effective for reducing the time duration of learning transients after start-up, which turns out to be tipically quite long in UASC based on V_i^S [BBMT09].

Finally notice that, as a consequence of next lemma, safe operation with V_i^{RC} is restricted netiher to a compact uncertainty set $\mathscr{P}$ nor to linear plants.

Lemma 3.3.1 – Consider the adaptive switched system $(P/C_{\sigma(\cdot)})$ in (2.6). Then, the test functionals $V^{\mathrm{RC}} := \left\{V_i^{\mathrm{RC}},\ i \in \underline{N}\right\}$, resulting from V_i^{RC} as in (3.35), along with the controller family $\mathscr{C}$, yield an $\mathcal{N}$-cost-detectable pair $(V^{\mathrm{RC}}, \mathscr{C})$.

Proof. See Appendix A.

3.4 Concluding Remarks

In this chapter, consideration has been given on how to on-line infer stability of a potential feedback-loop made up by a given candidate controller interconnected in feedback with an uncertain plant, while the latter is possibly driven by a different controller [KS01, HL01a, AD08]. Assuming that a nominal model M_i is associated to the candidate controller C_i in such a way that the resulting so-called *tuned-loop* (M_i/C_i) be stable, an answer is provided by sufficient conditions expressed in terms of percentage measures of discrepancy between the potential loop and the tuned one associated to the given candidate controller.

A number of possible multi-model UASC systems were considered, each one equipped with a different test functional (Table 3.1). Their features were comparatively analyzed mainly in terms of generality of applicability, stability inference and on-line implementability. Among the various conceivable test functionals, the one deriving from V_i^{RC} not only retains the desirable asymptotic properties of UASC [SWPS07] (it only requires that Problem Feasibility be satisfied in order to yield stability), but, being intrinsically based on an inference of stability test, can moderate the chance that destabilizing controllers be switched-on, and, consequently, dramatically reduce both the magnitude and time duration of "learning" transients after start-up.

As shown in next chapter, multi-model UASC based on V_i^{RC}, though conceptually hinging upon the *virtual reference* entity, only requires computation of *prediction errors* [Mos95], an advantageous feature from an implementational viewpoint in case v_i is not well-defined. We will also see that the positive assessment of stability robustness of UASC against the presence of possibly persistent noises/disturbances in the loop, is also enjoyed by multi-model UASC.

The proposed reference-loop identification architecture allows one to address goals of robust stability and performance, at the expense of dense model distributions. In practice, trade-offs between conflicting requirements of a moderate memory/computational load and high performance must be tackled. In this respect, as will be exemplified in Section 4.3, multi-model UASC, by embodying a reference-loop identification scheme, is less susceptible than MMASC of [Mor95, NB97] to poor transients and unsatisfactory schedule even in case of a coarse nominal model distribution.

The adopted approach only concerns time-invariant plants. However, a truly adaptive switching control methodology should enable the supervisor to handle also either slowly time-varying plants or infrequently abrupt plant changes. Preliminary results on this subject [BHMT09] indicate that the positive assessment of UASC stability robustness against the presence of possible plant time-variations is likely to be enjoyed by the proposed multi-model UASC scheme.

In this chapter, we focused the attention mainly on issues of model-based control vs. performance-based control. ASC schemes are, at times, also classified via

– Pre-Routing[Pre-Scheduling] Control vs. Non Pre-Routing[Pre-Scheduling] Control;

– Deterministic Control vs. Non Deterministic Control.

Multi-model UASC is a deterministic ASC scheme that does not require persistency of excitation, and, consequently, it has a wide-ranging potential in terms of generality of applicability. In this respect, alternative approaches to ASC can be found, *e.g.*, in [ND01, MA04, FAP06, ASS09]. Notice also that multi-model UASC does perform neither pre-routing nor pre-scheduling control. The rationale stems from the following observations: pre-routing [Nus83, Mar85, FB86] is designed to deal only with scheduling, and, hence, liable to exhibit violent and long-lasting transients; pre-scheduling [ZMF00, AM02] forces one to a priori specify a “worst-case” performance, which determines whether or not the switched-on controller must be replaced, making it problematic to address performance goals and, in practice, ruling out the possibility of combining switching with tuning.

Adaptive switching control has by now a rich literature spanning several diversified techniques for control design, stability/performance analysis, and applications. The monograph [Lib03] and the survey papers [KS01, HL01a, AD08] provide a detailed and unified overview of the topic.

Chapter 4

Multi-Model UASC Implementation

Having unveiled the advantages of multi-model UASC based on the V_i^{RC} identification functional, this chapter will be devoted to both analyze such a scheme in a noisy environment and exhibit efficient on-line computational architectures for general initial conditions. The main purpose, here, is to present the results when V_i^{RC} in (3.35) is computed via the l_2-norm, the most frequent norm adopted in ASC systems [Mor95, SWPS07, BBMT09], as well as to discuss the necessary modifications to V_i^{RC} for its on-line implementation in noisy environments. By all means, an extension to an arbitrary $\mathcal{N}$-norm, can be carried out by also exploiting the results that follow. Finally, for illustrative purposes, Section 4.3 discusses the two carts example of [GNL95], by which the benefits of embodying multiple nominal models in UASC are exhibited.

As beforehand, since crucial to improving UASC performance, it will be assumed throughout this chapter that the plant P in (2.6) be given by (3.13).

4.1 Nonzero Plant Initial Conditions

The switching methodology developed in Section 3.2 hinges upon the computation of $\tilde{z}_{i0}(\cdot)$. Apart form the "zero initial conditions" constraint, $\tilde{z}_i(\cdot)$ can be obtained by running, as shown in Fig. 3.1, the N tuned-loops (M_i/C_i) driven by $v_i(\cdot)$. The reason why this is impratical is twofold: first, computation of $v_i(\cdot)$ is impossible in case C_i is not causally stably invertible; and, second, even if $v_i(\cdot)$ is computable, it is advisable to simplify the implementation by avoiding to run the N tuned-loops (M_i/C_i). In view of those goals, an alternative and simpler way for computing $\tilde{z}_{i0}(\cdot)$

will be described next.

To this end, we make use of the first implication of (3.39). In this respect, notice that, under an $\mathscr{M}(\beta_{\mathrm{PC}})$ model distribution, $\beta_{\mathrm{PC}} < 1$, β_{PC} as in (3.24), one has

$$\|\Psi_{*/i}\|_{\mathrm{ind}} < \beta_{\mathrm{PC}} \qquad \Rightarrow \quad V_i^{\mathrm{RC}}(t) < \beta_{\mathrm{PC}}\,(1-\beta_{\mathrm{PC}})^{-1}, \qquad t \in \mathbb{Z}_+ \tag{4.1}$$

with V_i^{RC} as in (3.35), and $\Psi_{*/i}$ is the transfer matrix mapping z to $\tilde{z}_{i0}$,

$$\Psi_{*/i}(d) := \frac{[-S_i(d) \quad R_i(d)]'}{\chi_{i/i}(d)} \begin{bmatrix} \tilde{B}_i(d) & \tilde{A}_i(d) \end{bmatrix} =: Q_{i/i}(d)\tilde{L}_i(d)$$

Now, recall that

$$\begin{aligned} \tilde{z}_{i0}\,(t) &= \big(Q_{i/i}\,(d)\,[L_i\,(d) - L_*\,(d)]\big)_0\, z\,(t) \\ &= \tilde{\zeta}_i\,(t) - \zeta_{i*}\,(t) \end{aligned} \tag{4.2}$$

where $L_i\,(d) := [\,-B_i\,(d) \quad A_i\,(d)\,]$, $L_*\,(d) := [\,-B\,(d) \quad A\,(d)\,]$, and

$$\begin{aligned} \tilde{\zeta}_i\,(t) &= \big(Q_{i/i}\,(d)\; L_i\,(d)\big)_0\; z\,(t) \\ &= (Q_{i/i}(d))_0\, \epsilon_{i0}(t) \end{aligned} \tag{4.3}$$

$$\zeta_{i*}(t) \;=\; (Q_{i/i}(d)\, L_*(d))_0\, z(t) \tag{4.4}$$

The sequence $\tilde{\zeta}_i(\cdot)$ can be conveniently computed as shown next, while $\zeta_{i*}(\cdot)$, depending on P, is unknown but vanishes at an exponential rate as t increases. Consider first (4.3). There, $\epsilon_{i0}(t)$, $t \in \mathbb{Z}_+$, denotes the plant output prediction error at t based on M_i, *viz.*

$$\begin{aligned} \epsilon_{i0}(t) &:= (L_i(d))_0\, z(t) \\ &= A_i(d)y(t) - B_i(d)\delta u(t) \end{aligned} \tag{4.5}$$

computed as if the plant was at time 0 in "zero initial conditions", *viz.*

$$y(k) = \delta u(k) = 0, \quad k = -1, -2, \cdots \tag{4.6}$$

According to (3.21), $\epsilon_{i0}(t) = e_i(t) + A(d)\,y(t) - B(d)\,\delta u(t)$, hence $\epsilon_{i0}(t) = e_i(t)$, $t \geq \max\{\deg B, \deg A\}$. It is to be underlined that condition (4.6) need not to be satisfied by the actual plant I/O pairs, but it is only the appropriate fictitious initialization for (4.5) required by our needs. If the mentioned initialization coincides with the actual plant initial condition, $\tilde{z}_{i0}(t) = \tilde{\zeta}_i(t)$, because, in such a case, $\zeta_{i*}(t) = 0$ as follows by the fact that, under (4.6), $L_*(d)z(t) = 0$, $t \in \mathbb{Z}_+$.

Proposition 4.1.1 – *Consider the switched system $(P/C_{\sigma(\cdot)})$ under an arbitrary switching sequence $\sigma(\cdot)$. Let $\tilde{z}_{i0}(t)$ be the discrepancy data vector sequence between (P/C_i) and (M_i/C_i) both driven by the virtual reference $v_i(\cdot)$ resulting as if both (4.6) and $y_{i/i}(k) = \delta u_{i/i}(k) = 0$, $k = -1, -2, \cdots$ hold. Then, $\tilde{\zeta}_i(\cdot)$, the first additive component of $\tilde{z}_{i0}(\cdot)$ in (4.2), can be obtained, with no need of computing $v_i(\cdot)$, by using the difference equation $\tilde{\zeta}_i(t) = Q_{i/i}(d)\,\epsilon_{i0}(t)$, with $\epsilon_{i0}(t)$ the prediction errors (4.5), viz., $t \in \mathbb{Z}_+$*

$$\chi_{i/i}(d)\,\tilde{\zeta}_i(t) = [\,-S_i(d)\,R_i(d)\,]\,'\,\epsilon_{i0}(t) \tag{4.7}$$

from zero initial conditions:

$$\tilde{\zeta}_i(k) = \epsilon_{i0}(k) = 0, \quad k = -1, -2, \cdots \tag{4.8}$$

while z^t are the actual plant I/O data recorded from time 0 up to time t. $\square$

Remark 4.1.1 – Though $\tilde{\zeta}_i(\cdot)$ can be generated via the ficticious experiment depicted in Fig. 3.1 in which both (P/C_i) and (M_i/C_i) are driven by v_i, (4.7) circumvents the need of explicitly computing v_i in order to provide one with the sequence $\tilde{\zeta}_i(\cdot)$. Eq. (4.7) first appeared in [MA01] where it was pointed out that a significant advantage of (4.7) is that its validity holds true for general controllers C_i, even when, as in the case of non-stably invertible controllers, v_i is not well-defined.

Remark 4.1.2 – By (4.7), the numerator of (3.35) is seen to equal the square root of the energy of stably-filtered versions of prediction errors ϵ_{i0}. This is indeed the form of the test functionals adopted in all MMASC schemes à la Morse [Mor95, Hes98, HLM+01]. In fact, the multi-estimator à la Morse, in a discrete-time setting, reduces to compute stably-filtered prediction errors. However, despite such a similarity, there are strong differences between the multi-model UASC, based on (3.35), and MMASC. In this connection, in constrast with multi-model UASC, the filtering action on the prediction errors in MMASC is neither i-dependent nor deriving from *explicit* robust stability considerations. In connection with the filtering action provided by (4.7), the reader is referred to [MA01] for frequency-domain interpretations, and to [Gev93, Gev02] for connections with identification for control. However, the difference responsible for the decisive improvement of multi-model UASC over MMASC, is the quantity at the denominator of (3.35) which makes the test functional of multi-model UASC more elaborated than those, such as the one of MMASC, purely based on energies of stably-filtered versions of prediction errors. The role played of such a denominator is basically twofold: 1) As a data-dependent normalization term which removes possible biases on the indices i's [MA01]; and, 2) As a quantity which makes, for all switched-off idle indices, the ratio (3.35) bounded under all circumstances, even if the data $z(\cdot)$ are unbounded. This is an important feature for ensuring falsification of destabilizing controllers in the loop whenever unstable modes are excited. This has no counterpart in MMASC à la Morse where unbounded data typically yield unbounded test functionals, and, consequently, no guaranteed falsification of destabilizing controllers in the loop even when unstable modes be excited.

Lemma 4.1.1 – Consider the switched system $(P/C_{\sigma(\cdot)})$, with P as in (3.13), under an arbitrary switching sequence $\sigma(\cdot)$. Let P be in a compact uncertainty set $\mathscr{P}$. Then, $\zeta_{i*}(t)$, the unknown additive component of $\tilde{z}_{i0}(t)$ in (4.2), is bounded by an exponentially vanishing quantity as follows

$$|\zeta_{i*}(t)| \leq \Xi \left\|z^{n-1}\right\|_2 \lambda^{\underline{t}} \tag{4.9}$$

where: $\Xi = 0$, if (4.6) holds for the actual I/O pairs; otherwise, for generic plant initial conditions, $\Xi = \Xi(P, C, M)$ depends on the magnitude of P, the elements of C and M; n is the maximum among the orders of the irreducible transfer functions of $P \in \mathscr{P}$; λ is the maximum among the spectral radii of (M_i/C_i), $i \in \underline{N}$; $\underline{t} := \max\{t-(n-1), 0\}$.

Proof. Let $\epsilon_{*0} := (L_*(d))_0 z(t)$. Then, as $z(t)$ are plant I/O pairs, from definition of $L_*(d)$ in (4.2), one finds $\epsilon_{*0} = 0$, $t \geq n$. Let,

$$\begin{array}{ccc} \dfrac{[-S_i(d) \quad R_i(d)]'}{\chi_{i/i}(d)} & = & \displaystyle\sum_{k=0}^{\infty} [-s_{i,k} \quad r_{i,k}]' \, d^k \\ |s_{i,k}| \leq \bar{s}_i \lambda_i^k & & |r_{i,k}| \leq \bar{r}_i \lambda_i^k \end{array} \tag{4.10}$$

with $\lambda_i \in [0, 1)$ the spectral radious of (M_i/C_i). Thus,

$$\begin{aligned} |\zeta_{i*}(t)|^2 &= [s_{i,t-(n-1)} \; \epsilon_{*0}(n-1) + \cdots + s_{i,t} \; \epsilon_{*0}(0)]^2 + \\ &\quad\; [r_{i,t-(n-1)} \; \epsilon_{*0}(n-1) + \cdots + r_{i,t} \; \epsilon_{*0}(0)]^2 \\ &\leq M_i^2 \left\|\epsilon_{*0}^{n-1}\right\|_2^2 \lambda_i^{2\underline{t}} \\ &\leq M_i^2 \, |\mathscr{P}|^2 \left\|z^{n-1}\right\|_2^2 \lambda_i^{2\underline{t}} \end{aligned} \tag{4.11}$$

where: $M_i^2 = (1-\lambda_i^{2n})(\bar{s}_i^2 + \bar{r}_i^2)/(1-\lambda_i^2)$; $|\mathscr{P}|^2 = \max_{P \in \mathscr{P}} \left[\sum_{j=0}^{\deg A} a_j^2 + \sum_{j=0}^{\deg B} b_j^2\right]$; $\mathcal{P}(d) = B(d)/A(d)$ with $A(d) = \sum_{j=0}^{\deg A} a_j d^j$ and $B(d) = \sum_{j=0}^{\deg B} b_j d^j$ and $B(d)/A(d)$ irreducible transfer function. Then, (4.9) follows if $\Xi = \max_{i \in \underline{N}} M_i \, |\mathscr{P}|$ and $\lambda = \max_{i \in \underline{N}} \lambda_i$. □

As can be easily checked, the following lemma directly follows from (4.9).

Lemma 4.1.2 – Let $t_0 \in \mathbb{Z}_+$, $t_0 \geq n$, and $\zeta|_{t_0}^t := \{\zeta(t_0), \cdots, \zeta(t)\}$, $t_0 \leq t$. Then, under the same notations and assumptions of Lemma 4.1.1,

$$\frac{\left\|\zeta_{i*}|_{t_0}^t\right\|_2}{\left\|z^t\right\|_2} \leq \Xi(\lambda) \; \lambda^{t_0} \tag{4.12}$$

where $\Xi(\lambda) := \lambda^{-(n-1)}(1-\lambda^2)^{-1/2} \; \Xi$. □

In order to replace (3.35) with a functional computable from actual plant data for any possible initial condition, and ensure to the resulting multi-model UASC scheme properties similar to the ones of Theorem 3.2.2, the functional in (3.35) is redesigned so as to take into account the effects of the unknown vector $\zeta_{i*}(t)$. To this end, suppose that the nominal model set is distributed in such a way that $\mathscr{M} = \mathscr{M}(\beta_{\rm PC})$, $\beta_{\rm PC} < 1$, $\beta_{\rm PC}$ as in (3.24). Let η be a positive real such that $(1+\eta)\,\beta_{\rm PC} < 1$. Next, find the least integer $t_0 \in \mathbb{Z}_+$ such that

$$\Xi(\lambda)\,\lambda^{t_0} \leq \eta\,\beta_{\rm PC} \tag{4.13}$$

Then replace (3.35) as follows [1]

$$V_i^{\rm RC}(t) := \begin{cases} \left|\tilde{\zeta}_i(t)\right| \overline{J}_i^{\,-1/2} & ,0 \leq t \leq t_0 - 1 \\ \max\left\{ \dfrac{\left\| \tilde{\zeta}_i\Big|_{t_0}^{\tau} \right\|_2}{\left\| z^{t_0-1} + (z-\tilde{\zeta}_i)\Big|_{t_0}^{\tau} \right\|_2},\ \tau = t_0, \cdots, t \right\} & ,\text{elsewhere} \end{cases} \tag{4.14}$$

where:

1 - $\overline{J}_i$ is a constant depending, *e.g.* as elaborated in detail in [MA01], on the magnitude of the mixed sensitivity of (M_i/C_i) (see Section 4.3);

2 - By definition, the second of (4.14) is set equal to zero, whenever $\|z^t\|_2 = 0$.

Remark 4.1.3 – Note that the denominator in the second of (4.14) equals zero if and only if $\|z^t\|_2 = 0$. In fact, latter equality implies $\left\|z^{t_0-1}\right\|_2 = 0$. In such a case, being $t_0 \geq n$, at time t_0 the plant is at zero initial conditions. Accordingly, at the first time t where $\|z^t\|_2 > 0$, find $z(t) = [\delta u(t) \neq 0 \quad y(t) = 0]'$ and $\tilde{\zeta}_i(t) = 0$. Consequently, at such a time the denominator in the second of (4.14) becomes greater than zero.

Theorem 4.1.1 – *Let $z = \{\delta u,\, y\}$ be the vector-valued sequence of I/O plant data and $\tilde{\zeta}_i$, $i \in \underline{N}$, the first additive component of $\tilde{z}_{i0}$ in (4.2) computed via (4.7) and (4.8). Let σ be the switching sequence resulting from (2.6) and the HSL (2.9), with P as in (3.13) – under zero noises/disturbances – and test functional (4.14). Then, under Problem Feasibility, for any initial condition and reference r, the switching stops in finite time and $(P/C_{\sigma(\cdot)})$ is l_2-stable. Further, under a model distribution $\mathscr{M}(\beta_{PC})$, $\beta_{PC} < 1$, β_{PC} as in (3.24), properties i) - iv) of Theorem 3.2.2 hold true over the time interval $\mathbb{Z}_{t_0} := \{t_0, t_0+1, \cdots\}$ with β_{RC} replaced by $\beta/(1-\beta)$, $\beta := (1+\eta)\,\beta_{PC}$.*

Proof. See Appendix A. □

[1] The argument of the denominator in the second of (4.14) is understood as

$$\left\{z(0), \cdots, z(t_0-1), \left(z - \tilde{\zeta}_i\right)(t_0), \cdots, \left(z-\tilde{\zeta}_i\right)(\tau)\right\}, \quad \tau \geq t_0$$

Remark 4.1.4 – As pointed out in Lemma 4.1.1, if the plant starts at time zero from zero initial conditions, $\tilde{\zeta}_i(t) = \tilde{z}_{i0}(t)$, $t \in \mathbb{Z}_+$. Thus, $\Xi(\lambda) = 0$ and, consequently, $t_0 = 0$. Therefore, in such a case (4.14) is the same as (3.35), the latter computed relatively to the l_2-norm.

4.2 Implementation in a Noisy Environment

In accordance with the previous section, next results intend solely to sum up the conclusions when V_i^{RC} is computed relatively to the l_2-norm. By all means, an extension to an arbitrary $\mathcal{N}$-norm, can be carried out by following the same constructions as in Theorem 2.2.1 and Theorem 2.4.1.

Induced gains to which V_i^{RC} is related	Reference in	Noises in	Stability achieved
$\mathcal{P}$	$\mathbb{S}$	$\mathcal{P}(\mathbb{Z}_+)$	$\mathcal{P}$
$l_{2,\lambda}$	$\mathbb{S}$	$l_\infty(\mathbb{Z}_+)$	l_∞

Table 4.1: Relationship between V_i^{RC} in (4.16) and stability

In accordance with Theorem 2.2.1, safe operation with V_i^{RC} computed relatively to the l_2-norm requires that noises/disturbances have finite energy, a prescription this, in contrast with the most frequent situation where n_u, n_y are in $l_\infty(\mathbb{Z}_+)$ but not in $l_2(\mathbb{Z}_+)$. By (A.6) [2], if (P/C_i) is a stable candidate loop with spectral radius strictly smaller than $\sqrt{\lambda}$, one has

$$\|\tilde{\zeta}_i^t\|_{2,\lambda} \le \delta + \alpha \, \|(z - \tilde{\zeta}_i)^t\|_{2,\lambda} + \gamma \left\| \left(\begin{bmatrix} n_u \\ n_y \end{bmatrix} \right)^t \right\|_{2,\lambda} \tag{4.15}$$

for some positive reals α, γ and δ [Dat93]. Therefore, in accordance with (4.15), V_i^{RC} in (4.14) can be conveniently modified as follows.

$$V_i^{\mathrm{RC}}(t) := \begin{cases} \left| \tilde{\zeta}_i(t) \right| \overline{J}_i^{-1/2} & , 0 \le t \le t_0 - 1 \\ \max \left\{ \dfrac{\left\| \tilde{\zeta}_i \big|_{t_0}^{\tau} \right\|_{2,\lambda}}{\left\| z^{t_0 - 1} + \varsigma_i \big|_{t_0}^{\tau} \right\|_{2,\lambda}}, \ \tau = t_0, \cdots, t \right\} & , \text{elsewhere} \end{cases} \tag{4.16}$$

where $\lambda \in (0, 1]$ is such that the spectral radius of (M_i/C_i), $i \in \underline{N}$, is strictly smaller than $\sqrt{\lambda}$ (by construction, such a condition is automatically satisfied if $\lambda = 1$), and

$$\varsigma_i\big|_{t_0}^{\tau} := \left\{ \max\left\{ \mu, \left| \left(z - \tilde{\zeta}_i \right)(t_0) \right| \right\}, \cdots, \max\left\{ \mu, \left| \left(z - \tilde{\zeta}_i \right)(\tau) \right| \right\} \right\}, \ \ \tau \ge t_0$$

[2]Notice that $L_*(d)z(t) = B(d)\delta n_u(t) + A(d)n_y(t)$, $t \ge \max\{\deg A, \deg B\}$.

where $|\cdot|$ denotes Euclidean norm, and $\mu > 0$.

Let $\lambda = 1$. Under Problem Feasibility, (4.15) is valid. Consequently, in case $n_u, n_y \in \mathcal{P}(\mathbb{Z}_+)$, as $\| \varsigma_i|_{t_0}^{\tau} \|_2 \geq \mu\,(\tau + 1 - t_0)^{1/2}$, $\tau \geq t_0$, the HSL Lemma holds and $(P/C_{\sigma(\cdot)})$ is $\mathcal{P}-$stable $-$ see the proof of Theorem 4.1.1 for details $-$.

Let $\lambda \in (0, 1)$. In case $\mathscr{C}$ contain at least one candidate controller C_i such that spectral radius of (P/C_i) is strictly smaller than $\sqrt{\lambda}$, (4.15) is valid. Thus, if $n_u, n_y \in l_\infty(\mathbb{Z}_+)$ the rightmost term of (4.15) is bounded by a positive real κ, while $\| \varsigma_i|_{t_0}^{\tau} \|_{2,\lambda} \geq \mu$, $\tau \geq t_0$. Consequently, the HSL Lemma holds and $(P/C_{\sigma(\cdot)})$ is $l_\infty-$stable $-$ again, see the proof of Theorem 4.1.1 for details $-$.

Corollary 4.2.1 $-$ Let $z = \{\delta u,\, y\}$ be the vector-valued sequence of I/O plant data and $\tilde{\zeta}_i$, $i \in \underline{N}$, the first additive component of $\tilde{z}_{i0}$ in (4.2) computed via (4.7) and (4.8). Let σ be the switching sequence resulting from (2.6) and the HSL (2.9), with P as in (3.13) and test functional V_i^{RC} as in (4.16). Then,

i) Let $\lambda = 1$. Under Problem Feasibility, for any initial condition, reference r, and noises/ disturbances in $\mathcal{P}(\mathbb{Z}_+)$, the switching stops in finite time and $(P/C_{\sigma(\cdot)})$ is $\mathcal{P}-$stable.

ii) Let $\lambda \in (0, 1)$ be such that the spectral radius of (M_i/C_i), $i \in \underline{N}$, is strictly smaller than $\sqrt{\lambda}$. If for every $P \in \mathscr{P}$, there exists a candidate controller C_i such that the spectral radius of (P/C_i) is strictly smaller than $\sqrt{\lambda}$, for any initial condition, reference r, and noises/disturbances in $l_\infty(\mathbb{Z}_+)$, the switching stops in finite time and $(P/C_{\sigma(\cdot)})$ is $l_\infty-$stable. $\square$

Corollary 4.2.1, along with Theorem 4.1.1, complete the extension of the model-free case analysis of Chapter 2 to the present model-based approach to UASC.

4.3 A Two Carts Example

Consider a discrete-time (dt) LTI plant P, obtained from a continuous-time LTI plant by feeding its input via a zero-order holder and sampling its output every $0.1s$. The continuous-time LTI plant $\mathcal{P}(s)$ is made up by two carts [GNL95], both having mass $m_1 = m_2 = 1\ kg$. The carts are mechanically coupled by a spring with uncertain stiffness $\gamma \in [0.25,\ 1.5]\ N/m$. The plant state-space description is as follows

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\gamma/m_1 & 0 & \gamma/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ \gamma/m_2 & 0 & -\gamma/m_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m_1 \\ 0 \\ 1/m_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + n_y \tag{4.17}$$

where n_y stands for a measurement disturbance.

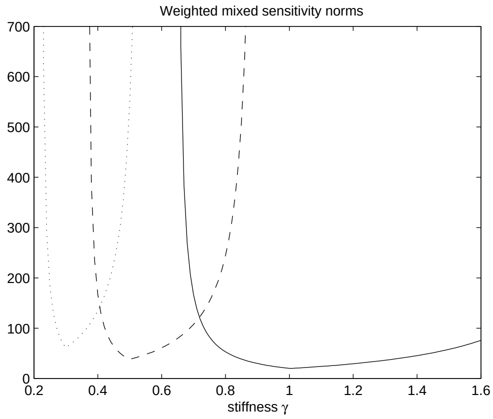
The aim is to control the position of one cart by applying a force to the other cart. Let $\mathcal{M}_\gamma(s) = B_\gamma(s)/A_\gamma(s)$ be the transfer function of the system with stiffness γ from u (force applied to the first cart) to y (second cart position) [3]. The transfer function of the corresponding tuned controller $\mathcal{C}^o_\gamma(s) = S^o_\gamma(s)/R^o_\gamma(s)$ was selected as the one among all stabilizing controllers $\mathcal{C}_\gamma(s) = S_\gamma(s)/R_\gamma(s)$ satisfying the weighted $\mathcal{H}_\infty$ mixed-sensitivity criterion [Kwa91]:

$$\mathcal{C}^o_\gamma(s) = \arg \inf_{\mathcal{C}_\gamma} \sup_{\omega} |V(j\omega)|^2 \left|\Sigma_{\gamma/\gamma}(j\omega)\right|^2$$

with $\Sigma_{\gamma/\gamma}$ the mixed sensitivity matrix of $(\mathcal{P}_\gamma/\mathcal{C}_\gamma)$ as in (3.5), and the weighting polynomial $V(s) = (s+2)^2(s^2 + \sqrt{2}s + 1)$ a strictly Hurwitz polynomial. Three different continuous-time controllers $\mathcal{C}_{\gamma_i}(s)$ were designed relatively to the nominal plant models $\mathcal{M}_{\gamma_i}(s)$ corresponding to the following three stiffness values: $\gamma_1 = 0.3$, $\gamma_2 = 0.5$ and $\gamma_3 = 1.0$ (see Table 4.2). Accordingly, the dt nominal models M_i and related dt candidate controllers C_i are the ones resulting from the use of an input zero-order holder with sampling time 0.1 s, and the subscript i corresponds to γ_i, $i \in \underline{3}$. The

[3] Notice that the system in (4.17) has already an integral action, *viz.*, for any γ, the discrete-time system in (3.2) resulting from (4.17) is such that $a(d)$ divides $1-d$. Accordingly, one can use u in place of δu, with no need of resorting to the plant incremental form (3.13). However, in order to adopt the same notational conventions considered so far, we shall set $a(d) := A(d)$, and $u(t) := \delta u(t)$ in the I/O representation (3.2) of the dt plant P.

Table 4.2: Weighted mixed sensitivity norms of $(\mathcal{P}_\gamma(s)/\mathcal{C}^o_{\gamma_i}(s))$ as a function of γ (γ_1 dotted line; γ_2 dashed line; γ_3 solid line).



$$\mathcal{C}^o_{\gamma_i}(s) = \arg\inf_{\mathcal{C}_\gamma}\sup_{\omega} |V(j\omega)|^2 \left|\Sigma_{\gamma_i/\gamma}(j\omega)\right|^2$$

Table 4.3: Controllers coefficients

	s_{i0}	s_{i1}	s_{i2}	s_{i3}	r_{i1}	r_{i2}	r_{i3}
C_1	43.4421	-128.9974	127.8710	-42.3118	-2.3876	1.9253	-0.5205
C_2	26.5098	-79.0064	78.7970	-26.2362	-2.3721	1.9023	-0.5118
C_3	13.2839	-40.3931	41.1406	-14.0267	-2.3223	1.8298	-0.4848

controllers coefficients are reported in Table 4.3. Each controller yields closed-loop stability for all values of γ in the following subintervals of the uncertainty interval:

$$\mathfrak{G}_1 = [0.25,\ 0.499)\ N/m, \quad \mathfrak{G}_2 = (0.361,\ 0.830)\ N/m, \quad \mathfrak{G}_3 = (0.631,\ 1.5]\ N/m$$

Experimental set-up $-$ In all simulations reported hereafter the hysteresis constant h is set equal to 0.01; the reference $r(t)$ is a square-wave of amplitude $\pm 2.5\ m$ and period 50 s, while n_y is taken to be Gaussian with zero mean and variance $\sigma_n^2 = 0.1$.
Throughout this section, attention will be focused on the following test functionals:

1) $V_i^M(t) := \|\epsilon_{i0}^t\|_{2,\lambda}$, where ϵ_{i0} is the prediction error in (4.5);

2) $V_i^S(t) := \max\left\{ \dfrac{\|(\,[w_i - y\ \delta u]\,)^\tau\|_{2,\lambda}}{\|\nu_i^\tau\|_{2,\lambda}},\ \ \tau = 0, \cdots, t \right\}$, *viz.* the model-free test functional as in (3.38), where $\nu_i^\tau =: \{\max\{\mu, |w_i(0)|\}, \cdots, \max\{\mu, |w_i(\tau)|\}\}$; $\mu := 0.01$, and, being all the C_i's non-minimum phase, the modified virtual reference w_i of Remark 2.2.1 used in place of v_i;

3) $V^{\mathrm{U}}(t) =: \|\tilde{\zeta}_i^t\|_{2,\lambda}$, where $\tilde{\zeta}_i$ is as in (4.7);

4) $V^{\mathrm{RC}}(t)$ as in (4.16), with $\mu := 0.01$ and $t_0 := 10$ ($1s$).

In words, V_i^M is the test functional purely based on energies of unfiltered prediction errors, such as the one typically adopted in MMASC schemes [Mor95, NB97, HLM+01], while V_i^U is a V_i^M variant based on energies of stably filtered prediction errors (cf. (4.7)); finally, V_i^S is the model-free test functional adopted in UASC literature [ST97, SWPS07].

4.3.1 Part # 1: Plant-model matching/mismatching

We begin with selecting a set of representative values of γ: $\gamma = 0.35$ (only C_1 stabilizes P, but there is a large mismatch between M_1 and P); $\gamma = 0.64$ (both C_2 and C_3 stabilize P, but C_3 does not perform satisfactorily); $\gamma = 1$ (a plant-model matching case). Simulation comparisons are first made among V^M, V^S and V^{RC}, in both zero plant initial conditions ($x(0) = 0_4$) and nonzero plant initial conditions ($x(0) := x_0 = [1 - 0.5 - 0.6\ 0.7]'$) cases. Simulation results relatively to the HSL (2.9), with $\sigma(0) = 3$, are reported in Table 4.4 and depicted in Figs. 4.1-4.4.

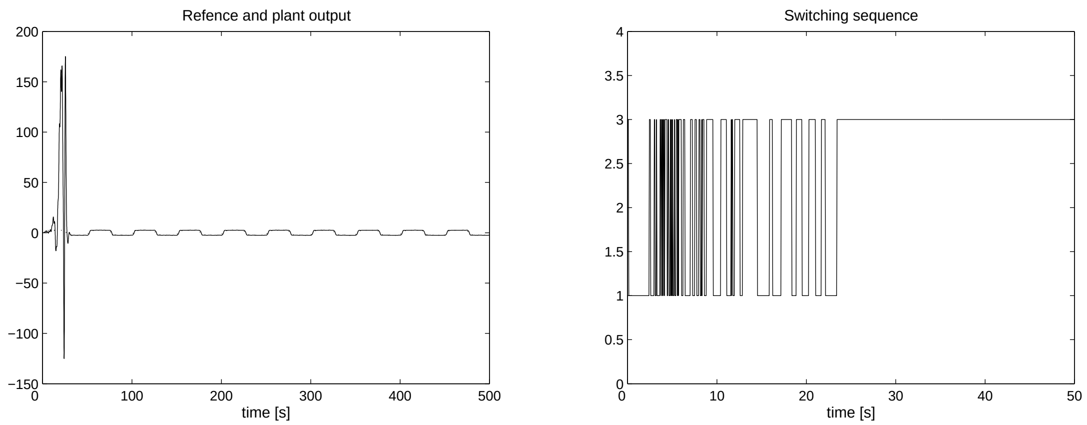


Figure 4.1: Simulation results for V^S, with $\gamma = 1$, $\sigma(0) = 3$, $\lambda = 1$, $x(0) = 0_4$. (a) Reference and plant output; (b) Corresponding switching sequence.

Some comments are in order. As expected, in model-free UASC, destabilizing controllers are repeatedly switched-on in the loop over a long initial "learning" interval (before the final controller is eventually switched-on), making the switched system "unstable" in a practical sense. This happens irrespective of λ and also in case the plant P matches one of the nominal models M_i's (case $\gamma = 1$), the initial controller corresponds to the tuned one and the plant starts at time 0 from zero initial conditions. As it emerges from Table 4.4, in the just mentioned plant-model matching case, the initial transient is greatly reduced in both MMASC and multi-model UASC, as the stabilizing candidate controller C_3 is held in the loop and never switched-off.

However, as for MMASC, performance deteriorates whenever a significant plant-model mismatch is present. *E.g.*, consider $\gamma = 0.35$. As it emerges from Table 4.4, MMASC takes much larger time to eventually switch-on to C_1 than multi-model UASC. As discussed next, there is more than one reason responsible for the decisive improvement of multi-model UASC over MMASC. Here we simply note that, in connection with stability, the key difference between V^{RC} and V^M is the quantity at the denominator of V^{RC} which makes, for all switched-off idle indices, the ratio (4.16) bounded under all circumstances, even if the data $z(\cdot)$ are unbounded. This is an important feature for ensuring, irrespective of possible plant-model mismatches, falsification of destabilizing controllers in the loop whenever unstable modes are excited. Such a feature has no counterpart in MMASC of [Mor95, NB97, HLM+01] where unbounded data typically yield unbounded test functionals (cf.

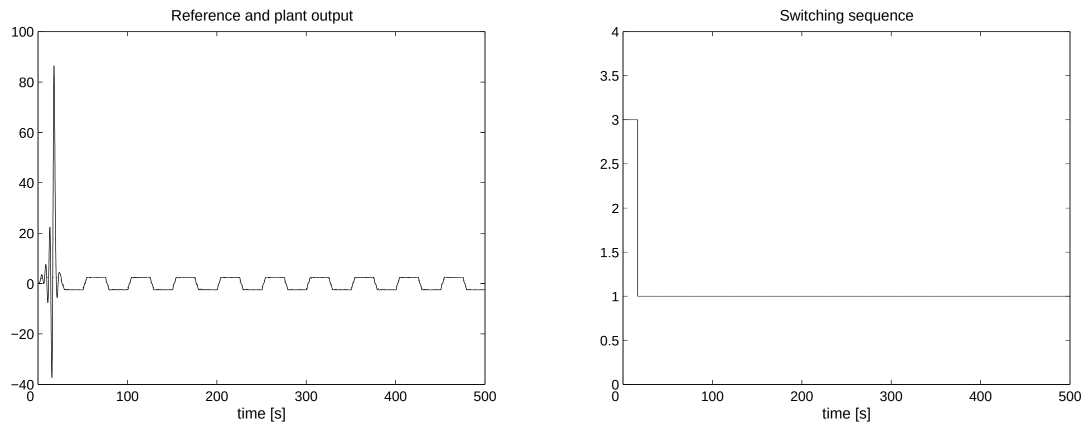


Figure 4.2: Simulation results for V^M, with $\gamma = 0.35$, $\sigma(0) = 3$, $\lambda = 1$, $x(0) = 0_4$. (a) Reference and plant output; (b) Corresponding switching sequence.

also Example B of Section 3.1). This observation is in fact confirmed by experimental evidence. In case $\gamma = 0.35$, although after an initial "learning" phase, both MMASC and multi-model UASC correctly sort the test functionals with respect to the suitability of the candidate controllers (and select C_1 as the final controller), such a transient is not "negligible" for MMASC. The performance improvement achieved by multi-model UASC over MMASC comes exactly from the fact that V_1^{RC} is not affected by the divergence trend of data caused by C_3 (Fig. 4.3(b)), which, in turn, implies the prompt falsification of C_3. As shown in Fig. 4.3(a), such a feature is not enjoyed by MMASC, as $V_1^M(\cdot)$ "follows" the trend to instability caused by C_3. It is indeed straightforward to verify that

$$\epsilon_{i0}(t) = A_i(d)\,(P(d) - M_i(d))\,\delta u(t) \tag{4.18}$$

which implies that, apart from the plant-model matching case, any trend to instability be reflected in a trend to unboundedness of V_i^M, irrespective of the stability of the loop (P/C_i). In cases where the number of models covering the uncertainty set $\mathscr{P}$ is not large enough, such a phenomenon becomes

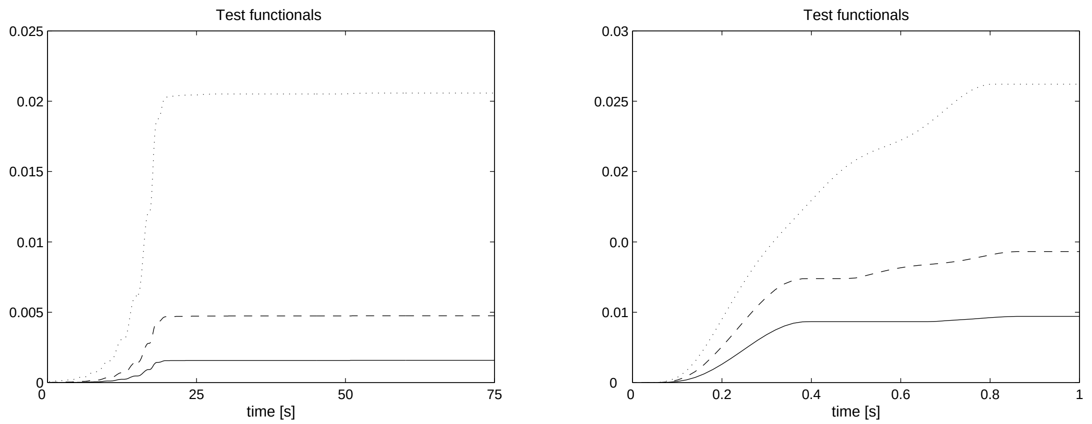


Figure 4.3: Simulation results for $\gamma = 0.35$, $\sigma(0) = 3$, $\lambda = 1$, $x(0) = 0_4$. (a) Test functional V^M; (b) Test functional V^{RC} ($V_1^M[V_1^{\mathrm{RC}}]$ solid line; $V_2^M[V_2^{\mathrm{RC}}]$ dashed line; $V_3^M[V_3^{\mathrm{RC}}]$ dotted line).

significant: For that reason, here, during the trend to instability exhibited in Fig. 4.2, ϵ_{10} remains comparable with ϵ_{30} for a quite long time interval (up to time $t < 16\,s$), which causes a much longer transient and a significant performance degradation (see also the simulation examples of [BBMT09]). In connection with Table 4.4, the worst-case simulation results for V^{RC} are depicted in Fig. 4.4.

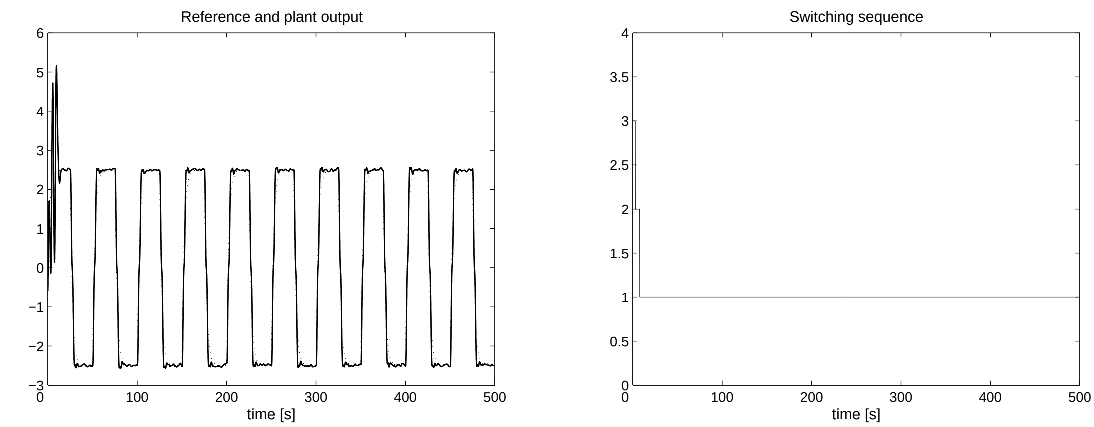


Figure 4.4: Worst-case simulation results for V^{RC}, with $\gamma = 0.35$, $\sigma(0) = 3$, $\lambda = 1$, $x(0) = x_0$. (a) Reference and plant output; (b) Corresponding switching sequence.

Table 4.4: Simulation comparisons among V^M, V^S and V^{RC} for different γ-values; controller index selected in accordance with the HSL (2.9), with $\sigma(0) = 3$.

	$\gamma = 0.35$			$\gamma = 0.64$			$\gamma = 1$		
	$\max_t \lvert y(t)\rvert$	C_f	t_f	$\max_t \lvert y(t)\rvert$	C_f	t_f	$\max_t \lvert y(t)\rvert$	C_f	t_f
$x(0) = 0_4$									
$\underline{\lambda = 0.95}$									
V^M	86.40	1	16.45 s	3.193	3	0 s	2.575	3	0 s
V^S	2.584	1	0.15 s	3.315	3	5.2 s	186.3	3	48.6 s
V^{RC}	2.592	1	0.40 s	2.671	2	1.25 s	2.575	3	0 s
$\underline{\lambda = 1}$									
V^M	89.02	1	16.35 s	3.212	3	0 s	2.575	3	0 s
V^S	2.605	1	0.15 s	3.209	3	10.3 s	175.1	3	23.9 s
V^{RC}	2.603	1	0.45 s	2.673	2	1.1 s	2.575	3	0 s
$x(0) = x_0$									
$\underline{\lambda = 0.95}$									
V^M	141.12	1	15.4 s	4.041	3	0 s	2.575	3	0 s
V^S	$> 10^3$	1	114 s	294.3	3	73.85 s	246.1	3	62.65 s
V^{RC}	4.109	1	5.15 s	4.059	2	2.75 s	2.575	3	0 s
$\underline{\lambda = 1}$									
V^M	$> 10^3$	1	34.4 s	4.049	3	0 s	2.575	3	0 s
V^S	$> 10^3$	1	88 s	294.1	3	71.05 s	246.1	3	0.4 s
V^{RC}	5.160	1	7.85 s	4.024	2	4.5 s	2.575	3	0 s

4.3.2 Part # 2: Filtering and normalizing actions

As noted in Remark 4.1.2, by (4.7), the numerator of V^{RC} (here denoted by V^U) is seen to equal the square root of (possibly weighted) energies of stably-filtered versions of prediction errors

ϵ_{i0}. Accordingly, the difference between V^{RC} and V^M consists of a filtering and a normalizing action of the prediction errors. As explained next, such an action turns out to be crucial not only for ensuring stability but also for improving the switching performance. In this respect, it is worth saying that, in MMASC literature, the main emphasis has been on the switching policy [Mor95, NB97, Hes98, HLM+01, HLM03a]. Hovewer, in general, no definite improvement can be achieved in ASC simply by means of alternative switching logics: Figs. 4.5-4.6 depict simulation results for V^M relatively to the *multiplicative* HSL [Hes98] with $h = 0.01$, and, respectively, *dwell-time* switching ($\tau_d := 5s$) [Mor95], in case $\gamma := 0.35$ and $\sigma(0) = 1$. From Figs. 4.5-4.6, one sees that, even if $\sigma(0) = 1$ (the stabilizing controller initially placed in the loop) and $x(0) = 0_4$, the schedule is far from satisfactory, and no convergence to C_1 is obtained.

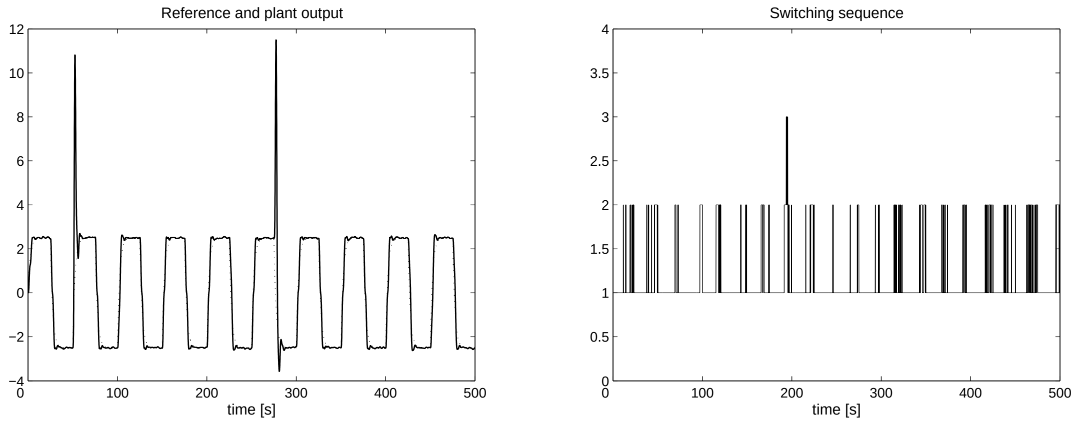


Figure 4.5: Simulation results for V^M, with $\gamma = 0.35$, $\sigma(0) = 1$, $\lambda = 0.95$, $x(0) = 0_4$ and multiplicative HSL. (a) Reference and plant output; (b) Corresponding switching sequence.

Some comments are in order. The filtering action is one of the two key elements responsible for the decisive improvement achieved by multi-model UASC over MMASC: First, in case where noises/disturbances are zero, by (4.2), one finds

$$\lim_{t\to\infty} \tilde{\zeta}_i(t) = \tilde{z}_{i0}(t), \qquad i \in \underline{N}$$

and, hence, V^U yields a perfect match between control and identification errors [4]. Even more importantly, the filtering action is such that, in contrast with (4.18), higher consideration in approximating P with M_i is given at those frequencies where the mixed sensitivity norm of (M_i/C_i) is larger, *viz.* frequencies where (M_i/C_i) is more vulnerable to disturbances [MA01]. Fig. 4.7 shows indeed that the simple filtering action embedded in the test functional V^U provides a noticeable performance improvement over V^M in terms of reduction of spurious switching.

[4] A similar use of filtering is in fact at the core of *Identification for Control* design [Gev93, dHS95, Gev02].

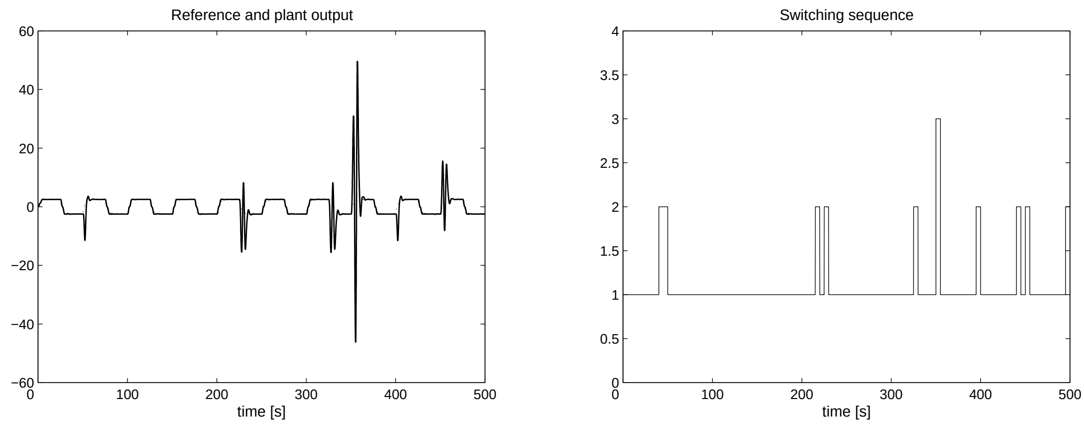


Figure 4.6: Simulation results for V^M, with $\gamma = 0.35$, $\sigma(0) = 1$, $\lambda = 0.95$, $x(0) = 0_4$ and dwell-time switching. (a) Reference and plant output; (b) Corresponding switching sequence.

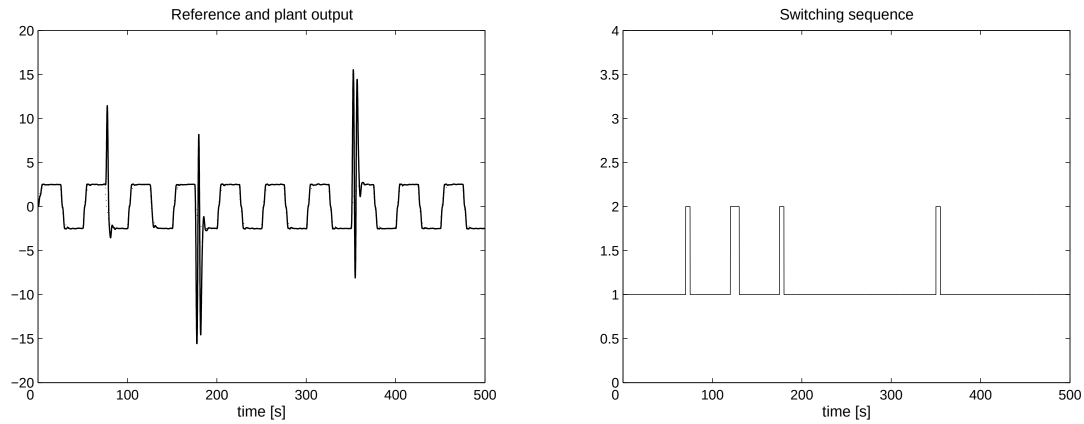


Figure 4.7: Simulation results for for test functional V^U, with $\gamma = 0.35$, $\sigma(0) = 3$, $\lambda = 0.95$, $x(0) = 0_4$ with dwell-time switching. (a) Reference and plant output; (b) Corresponding switching sequence.

The second key element responsible for the decisive improvement of multi-model UASC over MMASC, is the quantity at the denominator of (4.16) which makes the test functional of multi-model UASC more elaborated than those, such as the one of MMASC, purely based on energies of stably-filtered versions of prediction errors. The role played of such a denominator is twofold: As already noted, as a quantity which makes, for all switched-off idle indices, the ratio (4.16) bounded under all circumstances, even if the data $z(\cdot)$ are unbounded, and yields, in contrast with MMASC, guaranteed falsification of destabilizing controllers in the loop even when unstable modes are excited Apart from the guaranteed satisfaction of stability requirements, the denominator of (4.16) acts as a data-dependent normalization term which removes possible biases on the indices i's [MA01]. In fact − see also Remark 3.2.3 − R-percentage criteria in the normalized form (3.9)

$$\sigma := \arg\min_{i\in\underline{N}} \sup_{v_i\neq 0} \frac{\|(P/C_i)[v_i] - (M_i/C_i)[v_i]\|}{\|(M_i/C_i)[v_i]\|} \tag{4.19}$$

take into account the fact that the behavioral features of each tuned-loop (M_i/C_i) might be quite different at the various i's, especially if the uncertain plant dynamic range is large. This is an important feature which makes, from a practical viewpoint, R-percentage discrepancies more effective than MMASC for achieving a satisfactory switching performance. These observations also motivate the adoption of $|\,\tilde{\zeta}_i(t)\,|\,\overline{J}_i^{-1/2}$ in the first of (4.16): In view of (4.7), under ergodicity [Mos95] [5],

$$\begin{aligned}\lim_{t\to\infty}\frac{1}{t+1}\sum_{k=0}^{t}\tilde{\zeta}_i^2(k) &= E[\tilde{\zeta}_i^2(k)] = E[\{X_{i/i}(d)\epsilon_{i0}(k)\}^2] \\ &= E\left[\left\{A_i(d)\,X_{i/i}(d)[\tilde{P}_i(d)\delta u(k) + n_y(k)\right\}^2\right]\end{aligned} \tag{4.20}$$

with $\tilde{P}_i(d) = P(d) - M_i(d)$, $X_{i/i}(d) := N_i(d)/\chi_{i/i}(d)$, with $\chi_{i/i} := A_i(d)\,R_i(d) + B_i(d)\,S_i(d)$ the characteristic polynomial of (M_i/C_i), and N_i a solution of the spectral factorization problem [Mos95]: $N_i(d)N_i(d^{-1}) = R_i(d)R_i(d^{-1}) + S_i(d)S_i(d^{-1})$.

Hence, in connection with $|\,\tilde{\zeta}_i(t)\,|\,\overline{J}_i^{-1/2}$, one sees that, while $|\,\tilde{\zeta}_i(t)\,|$ acts as memoryless term (instrumental for achieving a satisfactory schedule in case of nonzero plant initial conditions), the normalizing term, first appeared in [MA01],

$$\overline{J}_i := \frac{1}{2}\int_{-\pi}^{\pi} |A_i(e^{j\,\omega})\,X_{i/i}(e^{j\,\omega})|^2\,d\omega \tag{4.21}$$

originates exactly from the fact that one might have different weights in (4.20) associated to each index i. This happens also in the present example, as $\overline{J}_1^{1/2} = 54.2727$, $\overline{J}_2^{1/2} = 33.5100$, and $\overline{J}_3^{1/2} = 17.6402$ [6]. In words, (4.21) and (4.22) indicate that, typically, without normalization, the mere filtering action of (4.7) provides no guarantees of entirely satisfactory controller selection. Fig. 4.8, which refers to dwell-time switching (the results with the HSL (2.9) and the multiplicative HSL are identical) confirms the foregoing analysis, as, by adopting V^{RC}, C_1 is never switched-off.

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[5] Here E denotes ensemble-average.

[6] The normalized filters $\overline{X}_{i/i}(d) := X_{i/i}(d)/\overline{J}_i^{-1/2}$ for the two carts example are as follows.

$$\begin{aligned}\overline{X}_{1/1}(d) &= \frac{-0.7477 + 2.322d - 2.409d^2 + 0.8348d^3}{1 - 6.382d + 17.45d^2 - 26.5d^3 + 24.14d^4 - 13.19d^5 + 4.004d^6 - 0.5206d^7} \\ \overline{X}_{2/2}(d) &= \frac{-0.7388 + 2.305d - 2.405d^2 + 0.8389d^3}{1 - 6.362d + 17.35d^2 - 26.28d^3 + 23.88d^4 - 13.02d^5 + 3.9444d^6 - 0.5119d^7} \\ \overline{X}_{3/3}(d) &= \frac{-0.7077 + 2.242d - 2.381d^2 + 0.8484d^3}{1 - 6.362d + 17.03d^2 - 25.59d^3 + 23.08d^4 - 12.49d^5 + 3.759d^6 - 0.4848d^7}\end{aligned} \tag{4.22}$$

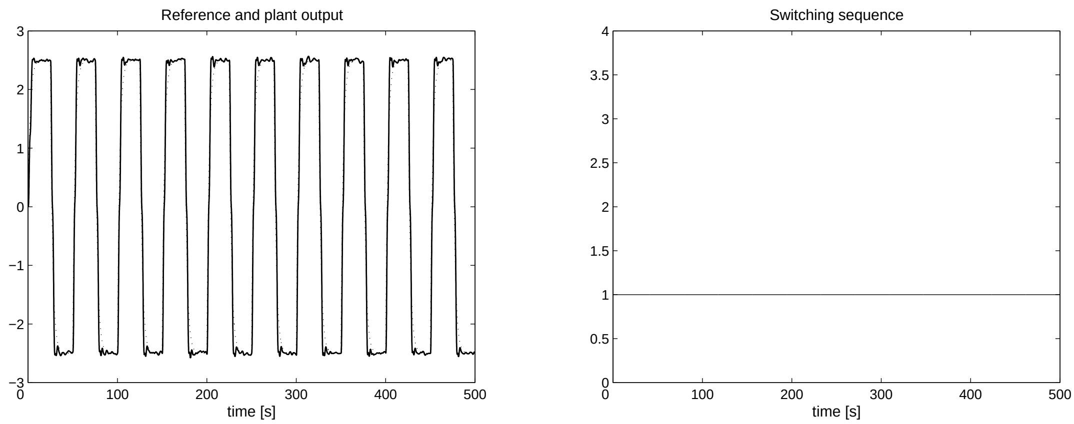

Figure 4.8: Simulation results for V^{RC}, with $\gamma = 0.35$, $\sigma(0) = 1$, $\lambda = 0.95$, $x(0) = 0_4$ and dwell-time switching. (a) Reference and plant output; (b) Corresponding switching sequence.

4.4 Concluding Remarks

In this chapter, we described relevant implementational issues and extended the conclusions derived in Chapter 3 to a noisy environment. In particular, Corollary 4.2.1, along with Theorem 4.1.1, complete the extension of the model-free case analysis of Chapter 2 and [ST95, ST97, SWPS07] to the present model-based approach to UASC. We also showed that the proposed scheme, though conceptually hinging upon the "virtual reference" entity, is on-line implementable without an explicit v_i-computation. This is an advantageous feature from an implementational viewpoint, as computation of (2.7) is numerically impossible in case of non-stably invertible controllers. As it emerges from the analysis, an interesting feature of multi-model UASC is that its implementation can be carried out by suitably filtering the prediction errors ϵ_i's. This establishes a conceptual link between the present approach and earlier MMASC contributions, since the multi-estimator of [Mor95, Mor96, NB97, HLM$^+$01], in a discrete-time setting, reduces to compute stably-filtered prediction errors. Motivated by such a fact, we provided in Section 4.1 and via numerical examples a closer comparison between multi-model UASC and MMASC as well as an interpretation of the filtering/normalizing action used in multi-model UASC in terms of frequency-domain analysis and connections with *Identification for Control* design [Gev93, dHS95, MA01, Gev02].

Appendix A

A.1 Control Implementation for Open-loop Unstable Controllers

The switching methodology developed in Chapter 2 hinges upon the multi-controller $C_{\sigma(t)}$ in (2.6). In principle, $C_{\sigma(t)}$ can be obtained by running, as shown in Fig. A.1, the N controllers driven by $(r-y)(\cdot)$. This means that, at every time, the supervisor selects one of the controllers in $\mathscr{C}$, while the other ones operate in stand-by mode.

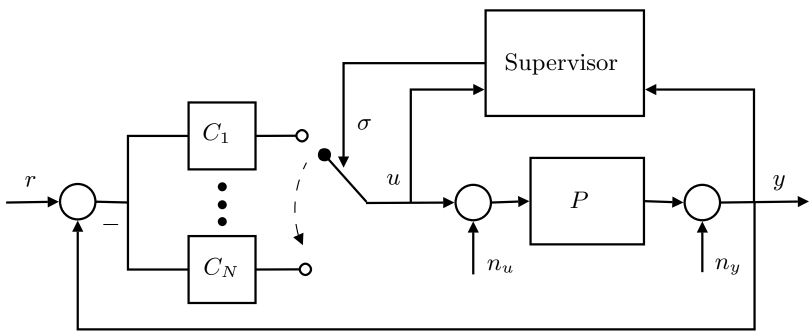

Figure A.1: Multi-controller architecture.

The reason why this is impractical is twofold: first, it is advisable to simplify the implementation by avoiding to run the N controllers C_i; even more importantly, in order to make computations numerically possible, it is required that each controller $-$ whether or not in charge of the plant $-$ run in a stable fashion. There are many ways to comply with the mentioned goals: two convenient architectures are briefly described next. Let each controller C_i be as in (2.5), with $r_i(d) = 1+\sum_{n=1}^{p} r_{in}d^n$ and $S_i(d) = s_{i0} + \sum_{n=1}^{p} s_{in}d^n$. A first possible solution, known as *State$-$Sharing*, consists of letting all controllers to share a common set of states and to switch the controller output gains [Mor95].

An example of a such an implementation consists of using as a shared state the vector

$$\chi(t) = [\epsilon(t-1), \cdots, \epsilon(t-p), u(t-1), \cdots, u(t-p)]'$$

where $\epsilon = r - y$. Then, over each interval of time where $\sigma(t) = i$ is constant, the control signal $u(t)$ is given by

$$C_i(r-y)(t) = [s_{i1}, \cdots, s_{ip}, -r_{i1}, \cdots, -r_{ip}]\, \chi(t) + s_{i0}\, \epsilon(t)$$

In this way, one avoids implementing each controller as a separate dynamical system, and $C_{\sigma(t)}$ reduces to a single controller with adjustable parameters.

A second conceivable alternative − here restricted to the case of CSCI controllers − is known as *Bumpless Control Transfer* [GGS01].

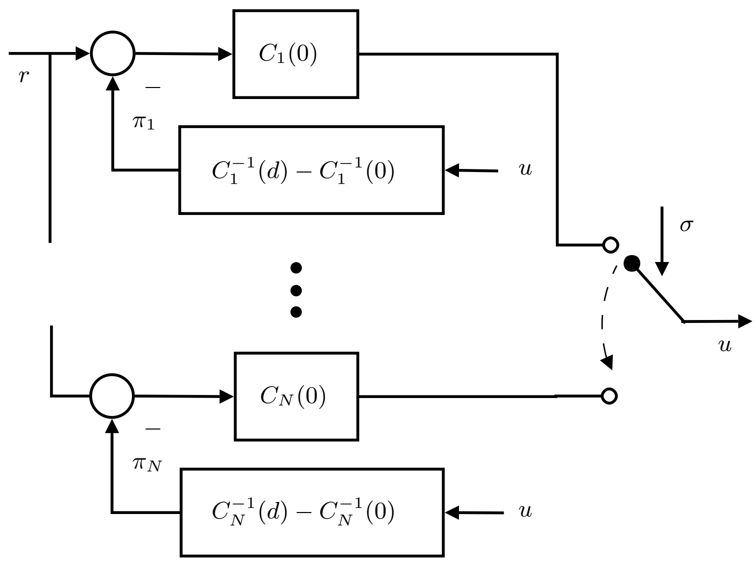

Figure A.2: Bumpless control transfer for CSCI controllers.

In the special case, when each C_i is CSCI (biproper and minimum phase), bumpless transfer between control can be achieved via the arrangement of Fig. A.2, corresponding to place each controller in the feedback form of anti-windup. In connection with the arrangment of Fig. A.2, it is simple to verify that each π_i is tied up to the corresponding virtual reference v_i in (2.7) via

$$v_i(t) = \pi_i(t) + y(t) + C_i^{-1}(0)\, u(t)$$

Consequently, when the i-th controller is switched-on at time t, one has

$$\begin{aligned} u(t) &= C_i(0)\,\epsilon(t) - C_i(0)\,\pi_i(t) \\ &= C_i(0)\,\epsilon(t) - [\,C_i(0) - S_i(d)\,]\;\pi_i(t) - r_i(d)\,u(t) + C_i^{-1}(0)\,S_i(d)\,u(t) \\ &= C_i(0)\;[\,\epsilon(t) - \epsilon_i(t)\,] + S_i(d)\,\epsilon_i(t) + [\,1 - r_i(d)\,]\;u(t) \end{aligned}$$

where $\epsilon_i = v_i - y$. Thus, one concludes that

$$r_i(d)\,u(t) = C_i(0)\;[\,\epsilon(t) - \epsilon_i(t)\,] + S_i(d)\,\epsilon_i(t)$$

In words, adopting the arrangment of Fig. A.2, amounts to resetting the state of C_i at the switch-on time as

$$\chi_i(t) = [\epsilon_i(t-1), \cdots, \epsilon_i(t-p), u(t-1), \cdots, u(t-p)]'$$

so that the switching transient of (P/C_i) turns out to be the same as the transient that would be observed with C_i permanently in the loop, with (P/C_i) driven by the virtual reference signal v_i (indeed the *virtual* closed-loop specified by (2.8)). A direct consequence of such a state-reinitialization is that $v_{\sigma(t)}(t) = r(t)$, $t \in \mathbb{Z}_+$. As beforehand, one can adopt a single dynamical system with adjustable parameters $-$ and state resetting upon switching $-$ rather than implementing each controller as a separate dynamical system as in Fig. A.2.

A.2 Proofs

Proof of Theorem 2.2.1

By construction, v_i can be viewed as the virtual reference that, if injected into the feedback system (P/C_i), would reproduce $z := \{u, y\}$. Then, provided that the disturbances n_u and n_y belong to $\mathcal{N}(\mathbb{Z}_+)$, for any index i corresponding to an $\mathcal{N}$-stable feedback system (P/C_i), there exist positive reals κ_1, κ_2, η_1 and η_2 such that

$$\|u^t\| \leq \kappa_1 \|v_i^t\| + \eta_1\,, \quad \|y^t\| \leq \kappa_2 \|v_i^t\| + \eta_2\,, \quad t \in \mathbb{Z}_+\,.$$

In other words, for indices corresponding to $\mathcal{N}$-stabilizing controller, $\mathcal{N}$-stability of the system (P/C_i) is always unfalsified by the I/O pair $\{v_i, z\}$, regardless of the switching sequence $\sigma(t)$, $t \in \mathbb{Z}_+$. As a consequence of cost-detectability, one concludes that $V_i(\cdot)$ remains bounded. Therefore, under problem feasibility, the HSL Lemma holds. Further, the test functional V_f, related to the final switched-on controller C_f, is bounded. This, along with the assumption B2, implies that $\mathcal{N}$-stability of (P/C_f) is unfalsified by $\{v_f, z\}$. Then, see also Def. 2.1.2, there exist finite nonnegative constants $\alpha_1, \alpha_2, \delta_1$ and δ_2 such that

$$\|u^t\| \leq \alpha_1 \|v_f^t\| + \delta_1\,, \quad \|y^t\| \leq \alpha_2 \|v_f^t\| + \delta_2\,, \quad t \in \mathbb{Z}_+$$

As the virtual reference v_f converges exponentially to the true reference r, there exists a finite nonnegative constant δ such that

$$\|v_f^t\| \leq \|r^t\| + \|v_f^t - r^t\| \leq \|r^t\| + \delta\,.$$

Consequently, one can conclude that

$$\|u^t\| \leq \alpha_1 \|r^t\| + \alpha_3\,, \quad \|y^t\| \leq \alpha_2 \|r^t\| + \alpha_4 \tag{A.1}$$

with $\alpha_3 := \alpha_1\delta + \delta_1$ and $\alpha_4 := \alpha_2\delta + \delta_2$. As (A.1) holds for the data from every possible input, $(P/C_{\sigma(\cdot)})$ turns out to be input-output stable with respect to the norm $\mathcal{N}$.

Proof of Theorem 3.2.1

For the sake of brevity, we prove the results relatively to V_i^{PC}, as the ones relatively to V_i^{PA} can be obtained along the same lines.

The switch stops in a finite time as a consequence of the following facts: first, each V_i^{PC} always takes on finite value whenever $\|z^t\| > 0$; second, each V_i^{PC} remain bounded over $\mathbb{Z}_+$ in view of stability of the tuned-loops (M_i/C_i)'s. In turn, $\mathscr{M}(\beta_{\mathrm{PC}})$ ensures that $V_f^{\mathrm{PC}}(t) \leq \beta_{\mathrm{PC}} + h < 1$, $t \in \mathbb{Z}_+$, f being the final controller index. Thus, (3.23) implies

$$\left\|z^t\right\| - \left\|z^t_{f/f}\right\| \leq \left\|\tilde{z}^t_{f/f}\right\| \leq (\beta_{\mathrm{PC}} + h)\, \left\|z^t\right\|$$

Notice that, regardless of the state of C_f is in at the final switching time t_f,

$$S_f\, v_f(t) = R_f\, \delta u(t) + S_f\, y(t) = S_f\, r(t), \quad t \geq t_f + p$$

with $p := \max\{\deg S_f,\, \deg R_f\}$, and

$$z_{f/f}(t) = (I_2 - Q_{f/f}\, L_f)\, z(t) = \frac{[A_f\ B_f]'}{\chi_{f/f}}\, S_f\, v_f(t) = \frac{[A_f\ B_f]'}{\chi_{f/f}}\, S_f\, r(t), \quad t \geq t_f + p$$

where $L_f = [-B_f\, A_f]$. Therefore, for some finite positive reals κ_1 and κ_2,

$$\left\|z^t\right\| \leq (1 - \beta_{\mathrm{PC}} - h)^{-1} \left\|z^t_{f/f}\right\| \leq \kappa_1 \left\|r^t\right\| + \kappa_2, \quad t \in \mathbb{Z}_+$$

where the last inequality follows from Hurwitzianity of $\chi_{f/f}$. We recall that, by Bezout identity [Mos95], $A(d)$ and $B(d)$ with strictly Hurwitz g.c.d. $D(d)$ amount to the existence of polynomials $X(d)$ and $Y(d)$ such that $X\,A + Y\,B = D$. This implies that $X\,a\,\delta u + Y\,a\,y = D\,u$, where $A(d) = a(d)(1-d)$. Consequently, $\mathcal{N}$-stability also holds in the sense (2.3) of Chapter 2 w.r.t. $\overline{z} = \{u,\, y\}$.

i) By the HSL (2.9) with test functionals (3.14), it can be seen that V_i^{PC}, every time a controller C_i is switched-on, increases at least by h. Now, there is always a β_{PC}-stabilizing controller, say of index $s \in S(P)$, such that

$$V_s^{\mathrm{PC}}(t) \leq \beta_{\mathrm{PC}}, \qquad t \in \mathbb{Z}_+$$

As a consequence, every index can be switched-on at most $\lceil \beta_{\mathrm{PC}}/h \rceil$-times. In fact, if it is switched-on one more time, its test functional would exceed at the switch-on time the upper bound of $V_s^{\mathrm{PC}}(t)$, contradicting (2.9).

ii) Let C_s be β_{PC}-stabilizing so that $V_s^{\mathrm{PC}}(t) \leq \beta_{\mathrm{PC}}$, $t \in \mathbb{Z}_+$. Thus, once (3.26) occurs, no destabilizing controller will be inserted in the loop at and after time t, since $V_s^{\mathrm{PC}}(\tau) < V_i^{\mathrm{PC}}(\tau)$, $i \in S^c(P)$, $\tau \geq t$.

iii) Let C_s be β_{PC}-stabilizing, and let (3.27) occur. Hence, at and after time t, only stabilizing controllers will be connected in feedback to the plant.

iv) Let a candidate controller be switched-on more than once. Then, its associated test functional becomes, once it is switched-on for the second time, equal to or greater than $h > \beta_{\mathrm{PC}}$. However, C_s with the β_{PC}-property implies $V_s^{\mathrm{PC}}(t) \leq \beta_{\mathrm{PC}}$. Consequently, if $h > \beta_{\mathrm{PC}}$ implies, in turn, that $V_s^{\mathrm{PC}}(t) < h$. Hence C_s is be never switched-off at and after time t.

Proof of Lemma 3.3.1

Note that, by letting $\{\delta u, y\}$ to be the I/O pair of interest, $\mathcal{N}$-cost-detectability in Def. 2.2.1 refers to (2.3) with $z = \{u, y\}$ changed into $z = \{\delta u, y\}$. As can be easily checked the same conclusions can be drawn relatively to $\{u, y\}$, *viz.* by letting $\tilde{z}_i := z - z_{i/i}$, with $z := \{u, y\}$ and $z_{i/i} := \{u_{i/i}, y_{i/i}\}$.

For brevity, let $V_i := V_i^{\mathrm{RC}}$. Suppose that there are finitely many switching times and let C_f be the final switched-on controller. According to Def. 2.2.1, cost-detectability of the pair $(V, \mathscr{C})$ holds provided that $V_f(\cdot)$ is bounded if and only if there exist finite reals $c_i \geq 0$, $i = 1, 2$, such that (2.3) holds. Suppose first that (2.3) holds. Then,

$$\frac{\left\| \tilde{z}_{f0}^t \right\|}{\| (z - \tilde{z}_{f0})^t \|} \leq 1 + \frac{\| z^t \|}{\| (z - \tilde{z}_{f0})^t \|} \leq 1 + \frac{c_1 + c_2 \, \|r^t\|}{\| (z - \tilde{z}_{f0})^t \|} \tag{A.2}$$

Recall that $V_f(t)$ is bounded for any t $-$ see the proof of Theorem 3.2.2 $-$. Further, as the denominator in (A.2) is monotonically non-decreasing, the rigthmost term of (A.2) diverges only if $\|r^t\|$ diverges as $t \to \infty$. Thus, it is only needed to show that the rightmost term of (A.2) remains bounded as $\|r^t\| \to \infty$. In order to see this, notice that

$$z - \tilde{z}_{f0} = (M_f/C_f)_0[v_f] = \frac{[S_f A_f \quad S_f B_f]'}{\chi_{f/f}} \, v_f \tag{A.3}$$

Since the virtual reference v_f related to the final constant controller exponentially approaches to the true reference signal r as $t \to \infty$, at sufficiently large t (A.3) yields

$$\|(z - \tilde{z}_{f0})^t\| \geq \mu \min_{\omega \in [0, 2\pi]} \frac{|S_f A_f| + |S_f B_f|}{|\chi_{f/f}|} \|r^t\| = \nu \, \|r^t\|$$

where the positive real μ, $\mu \in (0, 1]$, accounts for the truncation effects on the l_2-norm. Further, $\nu > 0$ because S_f is strictly Hurwitz [1] and A_f, B_f have strictly Hurwits g.c.d.'s. Hence, recalling (A.2), boundedness of $V_f(\cdot)$ follows at once.

Conversely, suppose that $V_f(\cdot)$ be bounded, *viz.* there exists a positive real κ such that $\left\| \tilde{z}_{f0}^t \right\| / \| (z - \tilde{z}_{f0})^t \| \leq \kappa$, $t \in \mathbb{Z}_+$. Then, by triangular inequality,

$$\|z^t\| \leq (1 + \kappa) \left\| (z - \tilde{z}_{f0})^t \right\| \tag{A.4}$$

Further, in view of (A.3), one has

$$\left\| (z - \tilde{z}_{f0})^t \right\| \leq \bar{\alpha} \, \|v_f^t\| + \bar{\beta} \tag{A.5}$$

for some positive reals $\bar{\alpha}$ and $\bar{\beta}$, being $\chi_{f/f}$ strictly Hurwitz. Thus, one obtains (2.3) by combining (A.4) with (A.5) and recalling that v_f converges to r.

Finally notice that, by exploiting the results of Chapter 4, one can prove that $\mathcal{N}$-cost-detectibility of V_i^{RC} also holds under arbitrary plant initial conditions as well as noises/disturbances belonging to $\mathcal{N}(\mathbb{Z}_+)$ − cf. (4.14) and (4.16) −.

Proof of Theorem 4.1.1

As can be checked, the mapping from $z - \tilde{\zeta}_i$ to $\tilde{\zeta}_i$ satisfies [2]

$$\tilde{\zeta}_i(t) = (\Omega_{*/i})_0 \, [z - \tilde{\zeta}_i](t) - (T_{*/i})_0 \, [z](t) \tag{A.6}$$

with $\Omega_{*/i}$ and $T_{*/i}$ as in (3.34), and, respectively, (3.37). Since $L_*[z] = 0$, $t \geq \max\{\deg A, \deg B\}$, under Problem Feasibility, $\|((T_{*/i})_0 \, [z])^t\|_2$ is finite for every $t \in \mathbb{Z}_+$. and the HSL Lemma holds.

[1] The notion of cost-detectability for non CSCI controllers can be found in [MCMS07]

[2] Of course, under zero initial conditions, the rightmost term of (A.6) equals zero, while $\tilde{\zeta}_i \equiv \tilde{z}_{i0}$.

Then, the test functional V_i^{RC} in (4.14), related to the final switched-on controller C_f is bounded, *viz.* there exists a positive real κ such that

$$\frac{\left\| \tilde{\zeta}_f \Big|_{t_0}^{t} \right\|_2}{\left\| z^{t_0-1} + (z - \tilde{\zeta}_f)\Big|_{t_0}^{t} \right\|_2} \leq \kappa, \quad t \in \mathbb{Z}_+$$

Then, by triangular inequality,

$$\|z|_{t_0}^t\|_2 \leq \kappa \, \|z^{t_0-1}\|_2 + (1+\kappa) \, \|(z-\tilde{\zeta}_f)|_{t_0}^t\|_2$$

Recalling (4.3), one finds that

$$\begin{aligned} z(t) - \tilde{\zeta}_f(t) &= z(t) - Q_{f/f} \, L_f \, z(t) = \\ &= \frac{[A_f \quad B_f]'}{\chi_{f/f}} \, (R_f \, \delta u(t) + S_f \, y(t)) \end{aligned} \tag{A.7}$$

Further, regardless of the state the controller C_f is in at time t_f, one has $R_f \, \delta u(t) + S_f \, y(t) = S_f \, r(t)$ for $t > t_f + p$ with $p = \max\{\deg S_f, \deg \, R_f\}$. This, along with the fact that $\chi_{f/f}$ is strictly Hurwitz, implies that $\|(z - \tilde{\zeta}_f)^t\|_2 \leq \bar{\alpha} \, \|r^t\|_2 + \bar{\beta}$ for some positive reals $\bar{\alpha}$ and $\bar{\beta}$. Hence, l_2-stability follows at once.

As for properties i) - iv), consider, $t \in \mathbb{Z}_{t_0}$, that there is always a β_{PC}-stabilizing controller, say of index $s \in S(P)$, such that

$$\Lambda_s(t) \leq \left(\left\| \tilde{\zeta}_s \Big|_{t_0}^{t} \right\|_2 / \, \|z^t\|_2 \right) \left(1 - \left\| \tilde{\zeta}_s \Big|_{t_0}^{t} \right\|_2 / \, \|z^t\|_2 \right)^{-1} \tag{A.8}$$

where $\Lambda_s(t)$ denotes the argument of the second of (4.14), and

$$\frac{\left\| \tilde{\zeta}_s \Big|_{t_0}^{t} \right\|_2}{\|z^t\|_2} = \frac{\left\| (\tilde{z}_{s0} + \zeta_{s*})|_{t_0}^t \right\|_2}{\|z^t\|_2} \leq (1+\eta) \, \beta_{\mathrm{PC}} \tag{A.9}$$

where last inequality follows from s being a β_{PC}-stabilizing index, Lemma 4.1.2 and (4.13). In view of (4.1), the conclusion is that the proof of Theorem 3.2.2 for i) - iv) carries over to Theorem 4.1.1, provided $t \in \mathbb{Z}_{t_0}$ and β_{RC} replaced by $\beta/(1-\beta)$, $\beta := (1+\eta) \, \beta_{\mathrm{PC}}$.

Part II

Predictive Switching Control of Linear Plants under Input Constraints and Persistent Disturbances

Chapter 5

Feasibility of Horizon-switching Predictive Control under Input Constraints

Control of input-saturated dynamic systems, though a fundamental issue in control engineering [BM95, GOMW00, GGS01], has been given exhaustive and constructive systematic answers mainly only during the last twenty years.

From one side it was characterized by the class of dynamical linear time-invariant (LTI) systems whose state can be asymptotically driven to zero with arbitrarily small controls [SSY94, Bla99, HL01b]: the so called ANCBI (Asymptotically Null-Controllable with Bounded Input) systems, which, in discrete-time, coincide with all stabilizable LTI systems with eigenvalues on the closed unit disk. Hence, they encompass stable systems with integrator chains of arbitrary complexity, and are representative of a great deal of processes of practical interest [Tee99]. From another side, linear control structures were shown to only provide semi-global stabilization of input-saturated ANCBI systems [Lin95, Lin98]. Non-linear state feedback schemes for input-saturated ANCBI plants were discussed in [SY91, SSY94]. However, such schemes amount to low-gain control strategies which feature poor regulation performance. In an attempt to provide enhanced performance, gain-scheduling variants, akin to the approach adopted in [Mos05] and the present one, were proposed in [ARS96, LSS96]. A third source of contributions to the topic has been originated within both model-based predictive control (MBPC) [GPM89, Mos95, MRRS00] and anti-windup control schemes [HKH87, Tee92, GOMW00, GHP$^+$03] whereby suitable corrections to a pre-designed compensator are generated whenever input saturations take place. To date, little attention has been devoted on how to deal with persistent disturbances of unknown arbitrary magnitude, a situation the latter under which MBPC appears very computationally demanding [SM98].

In a recent paper [Mos05], a computationally affordable solution to the pure regulation problem of discrete-time input-saturated LTI systems subject to persistent bounded disturbances of arbitrary magnitude has been proposed. The algorithm there proposed enjoys the following features:

1. It is realized via a supervisory switching control scheme whereby a feedback-gain, selected from a finite family of pre-designed candidate feedback-gains, is at any time switched-on in feedback to the plant according to the previous feedback-gain and the information, either complete or partial, on the current plant state;

2. No disturbance upper-bound need to be known;

3. The feedback-gain selection is made in accordance with a predictive control philosophy, and each candidate feedback-gain is tuned on to a different control-horizon;

4. The supervisory switching logic is flexible enough so as to enable the designer to simplify the scheme by trading off performance vs. memory and/or computational complexity, while retaining guaranteed stability properties.

Part II of this thesis aims at adapting the approach of [Mos05] to the case of set-point tracking for LTI systems subject to joint positional and incremental input saturations and also affected by persistent disturbances of unknown arbitrary magnitude. It is worth saying that, while positional input saturations have attracted a great deal of interest, fewer results apply to incremental input saturations. Incremental input saturations are a serious challenge in many automatic control applications, *e.g.* [LPBS97], joint position and rate saturated control [Lin98, TB97], flight control [Dor92, Len90]. In particular, it is known that they can induce significant destabilizing effects due to delay and phase-lag [BHSB96].

The main result of this chapter is the statement referred to as the *Feasibility Property*. Such a result will serve as a basis for the developments of next chapter.

5.1 Overall Problem

The overall problem around which Part II of this thesis is centered is the regulation problem of a discrete-time input-saturated LTI ANCBI plant, called the *Positional System* (PS)

$$\begin{cases} x(t+1) &= \Phi x(t) + Gu(t) + \xi(t) \\ y(t) &= Hx(t) + \zeta(t) \end{cases} \tag{5.1}$$

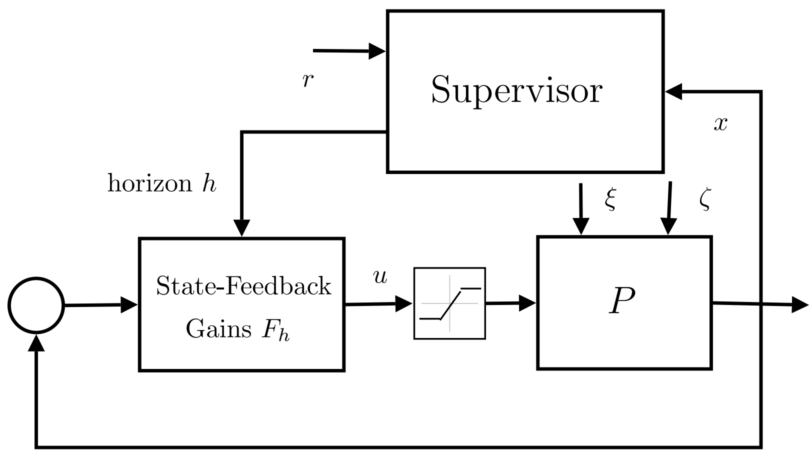

Figure 5.1: Horizon-switching Predictive control scheme.

with

$$\Sigma = (\Phi, G) \qquad \text{reachable}^1 \tag{5.2}$$

where Φ has all its eigenvalues of modulus less than or equal to one with arbitrary multiplicities; $t \in \mathbb{Z}_{-1} := \{-1, 0, 1, \cdots\}$; state $x \in \mathbb{R}^n$; input $u \in \mathbb{R}^m$; output $y \in \mathbb{R}^m$; ξ and ζ are exogenous disturbances. The plant input $u(t)$ and its increments $\delta u(t) := u(t) - u(t-1)$, $t \in \mathbb{Z}_+$ are subject to the following saturation constraints

$$u(t) \in \mathscr{U} := \{u \in \mathbb{R}^m : \quad U^- < u < U^+\} \tag{5.3}$$

$$\delta u(t) \in \mathscr{D} := \{\delta u \in \mathbb{R}^m : \quad \Delta^- < \delta u < \Delta^+\} \tag{5.4}$$

where $\Delta^- := [\Delta_1^-, \ldots, \Delta_m^-]'$ and $\Delta^+ := [\Delta_1^+, \ldots, \Delta_m^+]'$ with Δ_i^-, Δ_i^+, $i \in \underline{m}$, extended reals (similarly for u). Notice that the vector inequalities in (5.3) and (5.4) have to be interpreted in a component-wise sense. By the sake of simplicity [2], from now on, it will be assumed that $\overline{\Delta} = \Delta_i^+ = -\Delta_i^-$ and $\overline{U} = U_i^+ = -U_i^-$.

The problem of interest is to find, based on x (or a partial state-information), feedback controls

$$u = f(x) \tag{5.5}$$

which ensure global asymptotic stability and offset-free tracking in the presence of constant disturbances and set-point r, as well as finite l_∞-induced gain of the disturbance-to-state map from ξ to

[1]Note that (5.2) entails no loss of generality in that, if Σ is stabilizable but not reachable, all subsequent developments apply to the reachable subsystem of the plant obtained via a Gilbert-Kalman reachability decomposition.

[2]All the results that follow can be extended to the non-symmetric case via suitable technicalities.

x in (5.1)-(5.5) in the presence of time-varying disturbances and set-point. The approach adopted throughout this chapter and the next one consists of selecting a discrete family $\mathscr{F} = \{F_h\}_{h=1}^{\infty}$ of linear state-feedback gains F_h and a supervisory switching logic

$$h(t) = \ell(x(t), h(t-1)) \tag{5.6}$$

in such a way that the regulated plant have the stated stability properties.

Consider the following ANCBI system

$$x(t+1) = \begin{bmatrix} \Phi_s & 0 \\ 0 & \Phi_q \end{bmatrix} \begin{bmatrix} s(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} G_s \\ G_q \end{bmatrix} u(t) + \begin{bmatrix} \xi_s(t) \\ \xi_q(t) \end{bmatrix} \tag{5.7}$$

where: $\Phi := Diag\{\Phi_s\,, \Phi_q\}$, $G := [G_s'\, G_q']'$; $x = [s'\ q']'$; $dims = dim\Phi_s$; $\xi := [\xi_s'\, \xi_q']'$; Φ_s a stability matrix; and Φ_q has all its eigenvalues of unit modulus with arbitrary geometric multiplicities. It can be shown that a generic ANCBI system is algebraically equivalent to a system with transition-matrices having the block-diagonal structure in (5.7), with (Φ_q, G_q) reachable. Accordingly, for the sake of convenience, it will be assumed from now on that (5.1) be as in (5.7).

It is worth pointing out that (5.7) with $\xi_q(\cdot) \equiv 0$, has the most general structure in order to achieve state boundedness under saturated control and arbitrary l_∞-disturbances [HSLS98, SHS00b].

5.1.1 Incremental-model representation

According to the internal model principle [Mos95], a classic approach to track a reference r is to enforce an "integral action" from $e := y - r$ to u, r being the output reference. The related design hinges upon the so-called *incremental model* (IM) of (5.1)

$$\begin{cases} \chi(t+1) &= \mathcal{A}\,\chi(t) + \mathcal{B}\,\delta u(t) + \delta v(t) \\ e(t) &= \mathcal{C}\,\chi(t) + \delta w(t) \end{cases} \tag{5.8}$$

where $\chi(t) := [\delta x'(t)\ \ e'(t-1)]'$, $\delta x(t) := x(t) - x(t-1)$, $\delta v(t) := [\delta\xi'(t)\ \ \delta w'(t)]'$, $\delta w(t) := \delta\zeta(t) - \delta r(t)$, and

$$\mathcal{A} = \begin{bmatrix} \Phi & 0 \\ H & \mathrm{I}_m \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} G \\ 0_m \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} H & \mathrm{I}_m \end{bmatrix}. \tag{5.9}$$

It is well known that a linear state-feedback law $\delta u(t) = \mathscr{F}\chi(t)$, which stabilizes (5.8), yields an offset-free steady-state tracking error for the class of constant disturbances and references. As for (5.8), direct application of a PBH rank test [Kai80] shows that $(\mathcal{A}, \mathcal{B})$ is reachable if and only if (Φ, G) is such and

$$\det \begin{bmatrix} \mathrm{I}_n - \Phi & G \\ H & 0_m \end{bmatrix} \neq 0. \tag{5.10}$$

The latter is a necessary and sufficient condition for the existence of a stabilizing linear state-feedback with integral action for (5.1) under (5.2) [Dav75, Dav76].

5.2 Receding Horizon Feedback-Gains

Consider temporarily the noiseless unconstrained linear variant of (5.8). Because of properties that are motivated next in some detail, the candidate feedback-gains are as follows

$$F_h := -\Psi_u^{-1}\mathcal{B}'(\mathcal{A}^{h-1})'\mathscr{G}_h^{-1}\mathcal{A}^h \tag{5.11}$$

where: h is a positive integer, $h \geq \nu$, ν, $\nu \leq n$, the reachability index of $(\mathcal{A}, \mathcal{B})$; $\Psi_u = \Psi_u' > 0$; and $\mathscr{G}_h$ is the h-order reachability Gramian

$$\mathscr{G}_h := \sum_{k=1}^{h} \mathcal{A}^{k-1}\mathcal{B}\Psi_u^{-1}\mathcal{B}'(\mathcal{A}^{k-1})' \tag{5.12}$$

In connection with the incremental model (5.8), let χ be its state at time 0, and $\Omega_h(\chi)$ the set of all control increments ω of length h, $\omega = [\delta u'(0), \ldots, \delta u'(h-1)]'$, which drive the system state to the zero-state 0_χ in h time-steps

$$\Omega_h(\chi) := \left\{\omega \in (\mathbb{R}^m)^h : \chi(h) = 0_\chi\right\} \tag{5.13}$$

where $\chi(h) = \mathcal{A}^h\chi + \sum_{k=0}^{h-1}\mathcal{A}^{h-1-k}\mathcal{B}\delta u(k)$. Note that $\Omega_h(\chi) \neq \emptyset, \forall\chi \in \mathbb{R}^n$, $n := n_x + m$, if $h \geq \nu$. Furthermore, Let $\delta u_h(\chi)$ the element in $\Omega_h(\chi)$ of minimum energy

$$\sum_{k=0}^{h-1}\delta u'(k)\Psi_u\delta u(k) \;=\; \omega'\widehat{\Psi}_u\omega \tag{5.14}$$

where $\widehat{\Psi}_u := \underbrace{\mathrm{Diag}\left\{\Psi_u, \ldots, \Psi_u\right\}}_{h\text{-times}}$. Consequently, for $h \geq \nu$, $\delta u_h(\chi)$ is as follows

$$\begin{aligned}
\delta u_h(\chi) \;&:=\; [\delta u_h'(0|\chi), \ldots, \delta u_h'(h-1|\chi)]' \\
&=\; [\mathscr{F}_h'(0)\cdots\mathscr{F}_h'(h-1)]'\chi \\
&=\; \mathscr{F}_h\chi
\end{aligned} \tag{5.15}$$

$$\mathscr{F}_h \;:=\; -\widehat{\Psi}_u^{-1}\mathcal{R}_h'\mathscr{G}_h^{-1}\mathcal{A}^h \in \mathbb{R}^{m\,h\times n} \tag{5.16}$$

where $\mathcal{R}_h$ is the h-order reachability matrix

$$\mathcal{R}_h := \left[\mathcal{A}^{h-1}\mathcal{B}|\ldots|\mathcal{A}\mathcal{B}\,|\mathcal{B}\right] \tag{5.17}$$

Note that $\mathscr{F}_h(k)$ in (5.15) is given in terms of $\mathscr{F}_h$ as follows

$$\mathscr{F}_h(k) = \begin{bmatrix} 0_{m\times m\,k} & I_m & 0_{m\times m(h-1-k)} \end{bmatrix} \mathscr{F}_h \in \mathbb{R}^{m\times n}. \tag{5.18}$$

Hence, $F_h = \mathscr{F}_h(0)$ in (5.11) is recognized to be the feedback-gain matrix of the receding horizon regulator related to the *zero-terminal state* minimum energy control problem of horizon h [KP75, Mos95]. In the sequel, the integer h will be referred to as the *control horizon.*

Notice that for every $t \in \mathbb{Z}_+$

$$u_{h(t)}(k|\chi(t)) = u(t-1) + \sum_{k=0}^{h(t)-1} \delta u_{h(t)}(k|\chi(t)) \tag{5.19}$$

being $h(t)$ the control horizon at time t. The latter equation yields, at each iteration step, the control law for the system (5.1)

$$u(t) := u(t-1) + \delta u_{h(t)}(0|\chi(t)) \tag{5.20}$$

By letting $P(h) = (\mathcal{A}^h)'\mathscr{G}_h^{-1}\mathcal{A}^h$, it can be seen [Mos95] that, if $P(\nu) = (\mathcal{A}^\nu)'\mathscr{G}_\nu^{-1}\mathcal{A}^\nu$, then for $h \geq \nu$, $P(h+1)$ satisfies the Riccati difference equation

$$P(h+1) = \mathcal{A}'P(h)\mathcal{A} - \mathcal{A}'P(h)\mathcal{B}\left(\Psi_u + \mathcal{B}'P(h)\mathcal{B}\right)^{-1}\mathcal{B}'P(h)\mathcal{A} \tag{5.21}$$

and $F_{h+1} = -\left(\Psi_u + \mathcal{B}'P(h)\mathcal{B}\right)^{-1}\mathcal{B}'P(h)\mathcal{A}$. In addition, $P(h+1) \leq P(h)$, which implies that the following limit exists

$$\lim_{h\to\infty} P(h) =: P(\infty) \geq 0 \tag{5.22}$$

Further, if (5.1) is an ANCBI system, $P(\infty) = 0_{n\times n}$, with rate of convergence faster than or equal to $1/h$. Consider now

$$M_h(\chi) := \max\left\{ \frac{|[\delta u_h\,(k|\chi)]_i|}{\Delta}, \quad k+1 \in \underline{h};\ i \in \underline{m} \right\} \tag{5.23}$$

where $[\delta u]_i$ denotes the i-th component of the vector δu. Note that the whole sequence $\delta u_h(\chi)$ does not violate (5.4) if and only if $M_h(\chi) < 1$. By following the same lines as in [Mos05], one can prove that it is always possible to find a large enough horizon h so as to satisfy $M_h(\chi) < 1$.

Result [Mos05] $-$ *Let (5.8) be an* ANCBI *system, and $M_h(\chi)$ as in (5.23). Then,*

$$M_h(\chi) \leq \overline{M}\, h^{-1}\|\chi\| \tag{5.24}$$

where $\overline{M}$ is a positive real depending on $(\mathcal{A}, \mathcal{B})$. □

5.3 Feasibility of Horizon-switching Predictive Control

Consider next

$$\mathcal{M}_h(\chi) := \max\left\{\tilde{\alpha}\frac{|[\delta u_h(k|\chi)]_i|}{\overline{\Delta}}\ ,\ \alpha\frac{|[u_h(k|\chi)]_i|}{\overline{U}},\quad k+1\in\underline{h},\ i\in\underline{m}\right\}, \tag{5.25}$$

where $(\tilde{\alpha}=1,\alpha=0)$ corresponds to only incremental saturations; $(\tilde{\alpha}=0,\alpha=1)$ pertains to only positional saturations; $(\tilde{\alpha}=1,\alpha=1)$ to joint incremental and positional saturations. Next results shows that, given an arbitrary χ, there exist h such that $\mathcal{M}_h(\chi)<1$ for any of the possible pair $(\tilde{\alpha},\alpha)$. Consequently, one can always find a (virtual) input increment sequence (5.15) of large enough length h for which the saturation constraints (5.3) and (5.4) are jointly satisfied.

Consider the orthogonal decomposition

$$\mathbb{R}^n = \mathcal{R}((\mathcal{A}^n)')\oplus\mathcal{N}(\mathcal{A}^n), \tag{5.26}$$

where $\mathcal{N}(\mathcal{A}^n)=\mathcal{N}(\mathcal{A}^h)$, $\forall h\geq n=dim(\mathcal{A})$, and $\mathcal{R}(\cdot)$ and $\mathcal{N}(\cdot)$ denote range-space and, respectively, null-space. As, if $\chi^\perp\in\mathcal{N}(\mathcal{A}^n)$, $\delta u_h(k|\chi^\perp)=0$, $\forall h\geq n$, we can restrict the study to states in $\mathcal{R}((\mathcal{A}^n)')$. This amounts to assuming w.l.o.g. $\mathcal{A}$ non singular. Under such an assumption, the following properties hold.

Lemma 5.3.1 – Consider the incremental ANCBI model (5.8)-(5.10). Let $F_h=\mathscr{F}_h(0)$, with $\mathscr{F}_h$ and $\mathscr{F}_h(k)$ as in (5.16) and, respectively, (5.18). Then, the following properties hold: For every $k+1\in\underline{h}$,

$$\mathscr{F}_{h+1}(k)=\mathscr{F}_h(k)\left[I+O(h^{-1})\right] \tag{5.27}$$

If $\mathcal{A}_h:=\mathcal{A}+\mathcal{B}F_h$ and $l=[0'\ e']'\in\mathbb{R}^n$,

$$\mathscr{F}_h(k+1)=\mathscr{F}_h(k)\mathcal{A}_h\left[I+O(h^{-1})\right] \tag{5.28}$$

$$\mathscr{F}_h(k+1)l=\mathscr{F}_h(k)\left[I+O(h^{-1})\right]l \tag{5.29}$$

where $O(h^{-1})$ stands for a quantity of the order of h^{-1} or that vanishes at a faster rate.

Proof. From (5.15) one finds

$$\begin{aligned}\mathscr{F}_{h+1}(k) &= -\mathcal{B}'(\mathcal{A}^{h-k})'\mathscr{G}_{h+1}^{-1}\mathcal{A}^{h+1}\\ &= -\mathcal{B}'(\mathcal{A}^{h-1-k})'(\mathcal{A}'\mathscr{G}_{h+1}^{-1}\mathcal{A})\mathcal{A}^h\end{aligned} \tag{5.30}$$

Now, it is straightforward to verify that

$$\begin{aligned}(\mathcal{A}'\mathscr{G}_{h+1}^{-1}\mathcal{A})^{-1} &= \mathcal{A}^{-1}\mathscr{G}_{h+1}\mathcal{A}^{-T}=\mathscr{G}_h+\mathcal{A}^{-1}\mathcal{B}\mathcal{B}'\mathcal{A}^{-T}\\ &= \mathscr{G}_h(I+\mathscr{G}_h^{-1}\mathcal{A}^{-1}\mathcal{B}\mathcal{B}'\mathcal{A}^{-T})\end{aligned}$$

Therefore, one concludes that

$$\begin{aligned}\mathcal{A}'\mathscr{G}_{h+1}^{-1}\mathcal{A} &= (I+\mathscr{G}_h^{-1}\mathcal{A}^{-1}\mathcal{B}\mathcal{B}'\mathcal{A}^{-T})^{-1}\mathscr{G}_h^{-1} \\ &= \mathscr{G}_h^{-1}(I+O(h^{-1})) \end{aligned} \tag{5.31}$$

where, as shown in [Mos05], $\mathscr{G}_h^{-1} = O(h^{-1})$. Hence, (5.27) follows. The first equality in the next equation was shown [Mos05] to hold for the feedback-gains in (5.15), provided that $\mathcal{A}_h := \mathcal{A}+\mathcal{B}F_h$,

$$\mathscr{F}_h(k+1) = \mathscr{F}_{h-1}(k)\mathcal{A}_h = \mathscr{F}_h(k)\mathcal{A}_h\left[I+O(h^{-1})\right] \tag{5.32}$$

where the last equality follows from (5.27). By (5.28), the first equality in the following equation holds

$$\begin{aligned}\mathscr{F}_h(k+1)l &= \mathscr{F}_h(k)\mathcal{A}_h\left[I+O(h^{-1})\right]l \\ &= \mathscr{F}_h(k)\left[\mathcal{A}+\mathcal{B}F_h\right]\left[I+O(h^{-1})\right]l \\ &= \mathscr{F}_h(k)\left[I+O(h^{-1})\right]l \end{aligned} \tag{5.33}$$

where the last equality holds because $\mathcal{A}l = l$, and, by (5.24), $F_h = O(h^{-1})$. □

5.3.1 Feasibility under input constraints

In order to compute $\delta u_h(k|\chi)$ for large h, let

$$\chi = \chi(0) := l + v, \qquad l := \left[0' \quad e'(-1)\right]', \, v := \left[\delta x'(0) \quad 0'\right]'$$

Then, by linearity of $\delta u_h(k|\cdot)$ one has $\delta u_h(k|\chi) = \delta u_h(k|l) + \delta u_h(k|v)$. Further,

$$\begin{aligned}\delta u_h(k|l) &= \mathscr{F}_h(k)l \;=\; \mathscr{F}_h(k-1)\mathcal{A}_h\left[I+O(h^{-1})\right]l \\ &= \mathscr{F}_h(k-1)\left[I+O(h^{-1})\right]l \\ &= F_h\left[I+O(h^{-1})\right]l, \end{aligned} \tag{5.34}$$

where the second equality in (5.34) follows from (5.28), and the third from the fact that $\mathcal{A}_h l = l + O(h^{-1})$. Consequently,

$$\begin{aligned}u_h(k|\chi) &= u(-1) + \sum_{i=0}^{k}\delta u_h(i|\chi) \\ &= u(-1) + (k+1)F_h l + u_h(k|v) + O(h^{-1}), \end{aligned} \tag{5.35}$$

where $O(h^{-1})$, the rightmost term in (5.35), arises by taking into account that $F_h = O(h^{-1})$, and consequently $\sum_{i=0}^{k} F_h O(h^{-1}) l = O(h^{-1})$, $\; k+1 \in \underline{h}$.
Now, one must have

$$u_h(h-1|\chi) = u_\infty, \tag{5.36}$$

if u_∞ denotes the input vector to (5.1) which in steady-state yields the desired set-point r at the output of (5.1),

$$r = H(I - \Phi)^{-1}\left[Gu_\infty + \xi\right] + \zeta$$

Using (5.36) in (5.35), one finds $F_h l = \left[u_\infty - u(-1)\right]h^{-1} - u(h-1|v)h^{-1} + O(h^{-2})$. Therefore, $k+1 \in \underline{h}$,

$$\begin{aligned} u_h(k|\chi) &= u(-1) + \frac{k+1}{h}\left[u_\infty - u(-1)\right] + \\ &+ u_h(k|v) - \frac{k+1}{h}u_h(h-1|v) + O(h^{-1}) \end{aligned} \tag{5.37}$$

We now turn to show that $\delta u_h(k|v) = O(h^{-1})$ and, similarly, $u_h(k|v) = O(h^{-1})$. In fact,

$$\begin{aligned} \delta u_h(k|v) &= \mathscr{F}_h(k)v = \mathscr{F}_h(k-1)\mathcal{A}_h\left[I + O(h^{-1})\right]v \\ &= F_h\mathcal{A}_h^k\left[I + O(h^{-1})\right]v \end{aligned} \tag{5.38}$$

where the second equality follows from (5.28). That $\delta u_h(k|v) = O(h^{-1})$ follows from the fact that $F_h = O(h^{-1})$ and that $\mathcal{A}_h$ is a stability matrix. Using these two properties, it is easy to see that also $u_h(k|v) = O(h^{-1})$. Summing up, for every $k+1 \in \underline{h}$,

$$\delta u_h(k|\chi) = F_h\mathcal{A}_h^k\chi + O(h^{-2}), \tag{5.39}$$

$$u_h(k|\chi) = u(-1) + \frac{k+1}{h}\left[u_\infty - u(-1)\right] + O(h^{-1}). \tag{5.40}$$

Eq. (5.39) and (5.40) show that for any initial state $\chi \in \mathbb{R}^n$ it is always possible to find a large enough control horizon h so as to make the virtual input increments $\delta u_h(\cdot|\chi)$ and virtual inputs $u_h(\cdot|\chi)$ compatible with constrains (5.3) and (5.4) provided that $u(-1), u_\infty \in \mathcal{U}$. Notice that the latter property amounts to assuming that ξ, ζ and r are jointly within the input control range. Properties (5.39) and (5.40) are summed as follows [3].

Feasibility Property – *Consider the reachable* ANCBI *system (5.1) subject to joint positional and incremental input saturation constraints (5.3) and (5.4). Then, for every $\chi \in \mathbb{R}^n$ and in the presence of constant disturbances ξ, ζ and set-point r for which $u_\infty \in \mathcal{U}$, control horizons h can always be found so that $u_h(\chi) \in \mathcal{U}$ and $\delta u_h(\chi) \in \mathcal{D}$ provided that $u(-1) \in \mathcal{U}$.* □

5.3.2 Stability under arbitrary admissible switching

Taking into account the *Feasibility Property*, one can adopt as a switching logic for chosing h at each time t, call it $h(t)$. A control horizon sequence $\{h(t)\}_{t\in\mathbb{Z}_+}$, $h(t) \in \mathbb{Z}_+$, $h(t) \geq \nu$, is called *admissible* [Mos05] if

$$h(t+1) \geq h(t) - 1 \tag{5.41}$$

[3] It is worth pointing out that, in the absence of positional input saturations ($\alpha = 0$ in (5.25)), Feasibility Property directly follows from (5.24).

Let Σ denote the system (5.8) $-$ with zero disturbances and no input constraints $-$ under a time-varying control $\delta u(t) = F_{h(t)}\chi(t)$, $F_{h(t)} \in \mathscr{F}$,

$$\chi(t+1) = \mathcal{A} + \mathcal{B}\, F_{h(t)}\chi(t) =: \mathcal{A}_{h(t)}\,\chi(t)$$

Further, let $\mathscr{S}$ denote the set of all such admissible sequences, and $(\Sigma, \mathscr{S})$ the system (5.8) under an arbitrary admissible switching sequence in $\mathscr{S}$. The theorem that follows, first appeared in [Mos05], is crucial for our subsequent developments.

Theorem 5.3.1 *[Mos05] $-$ Consider the control system $(\Sigma, \mathscr{S})$ composed of the LTI reachable plant (5.8) under a time-varying state-feedback control $\delta u(t) = F_{h(t)}\chi(t)$ realized by arbitrary admissible switching sequences in $\mathscr{S}$. Then, provided that $\underline{h} \geq n$ and $\{h(t)\}_{t\in\mathbb{Z}_+}$ be bounded,* viz.

$$n \leq \underline{h} \leq h(t) \leq \overline{h} \tag{5.42}$$

with $\overline{h} < \infty$, $(\Sigma, \mathscr{S})$ is exponentially stable

$$\|\chi(t)\| \leq M\lambda^t \|\chi(0)\| \tag{5.43}$$

with $M \in (0, \infty)$, and decaying rate λ depending on $\overline{h}$ and $\underline{h}$. $\square$

5.4 Concluding Remarks

The main result of this chapter is the statement referred to as the *Feasibility Property.* A shown in next chapter, Feasibility Property, along with the admissibility condition (5.41), make it possible to exploit the stability results of Theorem 5.3.1 so as to to extend them, via appropriate switching logic supervisors $-$ such as the one introduced in [Mos05] $-$ to LTI systems subject to input constraints and persistent bounded disturbances of unknown arbitrary magnitude. It is worth saying that the control schemes deriving from (5.41), though of *predictive* nature, differ from classical predictive control schemes [GPM89, Mos95, GGS01], in that they do not solve at every step a mathematical programming problem. Another point is worth-mentioning. Notice that, there has been recently some interest [DM99, HM02] in finding rules which can ensure stability regardless of the algorithm used to orchestrate the switching. In the literature, such a mode of operation appears to be mainly motivated by the goal of enhancing performance. This is in particular the case for the switching logic schemes considered next, as the admissibility condition (5.41) establishes intrinsically the possibility of selecting the maximum regulation speed compatible with the saturation constraints. However, unlike [HM02] where stability is enforced via a suitable state-reinitialization upon switching, in the present schemes stability is ensured, thanks to the particular type of candidate feedback-gains, by simply imposing an admissibility condition on switching sequences. Such a feature allows one to develop switching logic supervisors which can be effective not only for enhancing performance but also when stability is at stake.

Chapter 6

Supervisory Control of Linear Plants under Input Constraints and Persistent Disturbances

In this chapter, predictive switching logic schemes are considered whereby a feedback-gain is switched-on at any time from a family of candidate feedback-gains so as to deal with the problem of set-point tracking in ANCBI systems under input constraints and persistent arbitrarily bounded disturbances. It is constructively shown that such schemes do exist which ensure global asymptotic and offset-free tracking in the presence of constant disturbances and set-point r, as well as finite l_∞ induced gain of the disturbance-to-state map in the presence of time-varying disturbances and set-point, while preserving the fulfillment of control-increment saturation constraints. In particular, Theorem 6.3.1 completes the extension of the approach of [Mos05] to the case of set-point tracking under input-increment saturations and persistent disturbances. Section 6.4 reports simulation examples which illustrate the effectiveness of the proposed technique. The results reported hereafter first appeared in [MT08, MTZ08a, MTZ08b].

6.1 Hysteresis Switching Logic

For convenience, let us rewrite the ANCBI positional system (PS) of interest

$$x(t+1) = \begin{bmatrix} \Phi_s & 0 \\ 0 & \Phi_q \end{bmatrix} \begin{bmatrix} s(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} G_s \\ G_q \end{bmatrix} u(t) + \begin{bmatrix} \xi_s(t) \\ \xi_q(t) \end{bmatrix} \tag{6.1}$$

where $x := [s' \, q']'$, $x \in \mathbb{R}^{n_x}$; Φ_s a stability matrix; and Φ_q has all its eigenvalues of unit modulus with arbitrary geometric multiplicities. The tracking error is given by

$$e(t) := y(t) - r(t)$$

where $y(t) = [H_s H_q] \, x(t) + \zeta(t)$, $y \in \mathbb{R}^m$ being the performance variable (output) and $r(t)$ the set-point to be tracked by the output. The vectors ξ_s, ξ_q and ζ represent arbitrary bounded distrurbances. Let δu be the PS input increment, computed as $\delta u(t) = F_{h(t)}\chi(t)$, where F_h is as in (5.11); $\chi(t) := [\delta x'(t) \, e(t-1)]'$, $\chi \in \mathbb{R}^{n_x+m}$, is the state of the incremental model (IM) (5.1) of the PS.

Consider the following *hysteresis switching logic* (HSL) ($\underline{h} \geq n$, $n = n_x + m$)

$$h(t) = \begin{cases} \tilde{h}(t), & \text{if } \mathcal{M}_{\tilde{h}(t)}(\chi(t)) < 1 \\ \hat{h}(t), & \text{otherwise.} \end{cases} \tag{6.2}$$

$$\tilde{h}(t) := \max\{\underline{h}, \, h(t-1) - 1\}$$

$$\hat{h}(t) := \min \{h \in \mathbb{Z}_+ : \quad h \geq h(t-1); \quad \mathcal{M}_h(\chi(t)) \leq 1 - \mu\}$$

where $t \in \mathbb{Z}_+$; $\mu \in (0, 1)$ is the *hysteresis constant*; $h(0) = \hat{h}(0)$, with $h(-1) = \underline{h}$, denotes the control horizon at time 0; and $\mathcal{M}_h(\chi)$ as in (5.25).

It is worth recalling that, in the absence of positional input saturations ($\alpha = 0$ in (5.25)), $\mathcal{M}_h(\chi)$ in (5.25) reduces to $M_h(\chi)$ in (5.23), for which (5.24) is valid.

Remark 6.1.1 – The supervisory logic (6.2) is responsible for choosing the horizon $h(t)$ given the state $\chi(t)$. In (6.2), $\underline{h}$, $\underline{h} \geq n$, denotes the minimum horizon whose choice is up to the designer (roughly, the larger $\underline{h}$, the narrower the frequency bandwidth of the closed-loop system with transition matrix $\mathcal{A} + \mathcal{B} F_{\underline{h}}$). Stability of the switched system is ensured by the admissibility condition $h(t) \geq h(t-1) - 1$. In words, the horizon is not allowed to decrease more than one unit at a single time-step, while arbitrary increases of the horizon do not destroy stability. The hysteresis constant μ makes h "easier" to decrease than to increase or stay constant. From a practical viewpoint, the switching logic (6.2) establishes intrinsically the possibility of selecting the maximum regulation speed compatible with the saturation constraints.

6.2 Constant Disturbances/Set-point Case

Next stability results, restricted to the case of constant disturbances and set-point, are a direct consequence of Feasibility Property and Theorem 5.3.1.

Theorem 6.2.1 – *Consider the ANCBI system (5.1) satisfying (5.2) and (5.10) and subject to joint positional and incremental input saturation constraints (5.3) and (5.4). Let its control increments* $\delta u(t) = F_{h(t)}\chi(t)$ *with* F_h *as in (5.11), and* $h(t)$ *chosen according to the HSL (6.2). Then, for every* $x(0) \in \mathbb{R}^{n_x}$ *and in the presence of constant disturbances* ξ_s, ξ_q *and* ζ*, the resulting switched system yields offset-free tracking of constant set-point* r *and global asymptotic stability provided that* $u(-1), u_\infty \in \mathcal{U}$ [1].

Proof. Assume that $h(0) = \hat{h}(0)$ exists finite. Thus, at the next time-step, $h(0) - 1$ is an admissible control horizon for the state $\chi(1) = \mathcal{A}_{h(0)}\chi(0)$ (the IM is indeed noise-free). Then, from (6.2), $\underline{h} \leq \hat{h}(1) \leq h(0) - 1$ and, hence, $h(1) = h(0) - 1$. In this way, $h(t)$ decreases by one at each time-step up to the time $t = h(0) - \underline{h}$ at which $h(t) = \underline{h}$. Thereafter, for $t \geq h(0) - \underline{h}$, $h(t)$ equals $\underline{h}$. As the resulting switching sequence is admissible and $h(t) \leq h(0)$, Theorem 5.3.1 ensures asymptotic stability. As the PS is ANCBI, (5.24) implies that $h(0) < \infty$ for any $x(0) \in \mathbb{R}^{n_x}$. □

Corollary 6.2.1 – Provided only input-increment saturations are present, under the same conditions as in Theorem 6.2.1, for every $x(0) \in \mathbb{R}^{n_x}$ and in the presence of constant disturbances ξ_s, ξ_q and ζ, the resulting switched system yields offset-free tracking of constant set-point r and global asymptotic stability. □

In Corollary 6.2.1, the absence of positional input constraints means that $\alpha = 0$ in (5.25). It is worth recalling that, in case $\alpha \neq 0$, if $(I - \Phi)$ is singular, no arbitrary bounded disturbances ξ_q can enter the integrators of the plant.

6.3 Horizon Resetting in the Time-varying Case

In case of non-constant disturbances and references signals, one cannot avail of the results of Theorem 5.3.1. As shown next, in the presence of time-varying disturbances and set-points, the proposed switching control scheme ensures l_∞ induced gain of the disturbance-to-state map while preserving the fulfillment of control-increment saturation constraints, *viz.* $\alpha = 0$ in (5.25). In that sense, next result complements the one of [Mos05], wherein predictive switching logic schemes based on the HSL (6.2) were introduced for solving the zero regulation problem of ANCBI systems subject to time-varying disturbances and positional input saturations, *viz.* $\tilde{\alpha} = 0$ in (5.25).

[1] Under the same condition as in Theorem 6.2.1, one also has global asymptotic stability for the class of disturbances and the reference sequences which become constant in finite time. Further, the switched system is also semi-globally exponentially stable (see [Mos05, MT08] for the necessary details).

6.3.1 Incremental model analysis

Before proceeding any further, some comments are in order. First, it is worth pointing out that the present problem of set-point tracking is subject to the same conceptual limitations as in the pure regulation problem case. Indeed, as can be checked, the time-varying disturbance $\delta\xi_q(t)$, entering the neutrally stable modes of $\chi(t)$ in (5.8), is generally not allowed to assume arbitrary values, *viz.* $\xi_q(t)$ cannot assume arbitrary incremental values [2]. Consequently, from now on, it will be assumed that the disturbance $\xi_q(t)$ in (5.1) be constant, *i.e.* $\xi_q(t) \equiv \xi_q$.
Another point is worth-mentioning. Seemingly, (5.8) is not in the form of a control-saturated LTI system for which it makes sense to consider stability and boundedness under arbitrary l_∞-disturbances. Hence, it is not possible to be sure that the direct adoption to the present case of the switching logic (6.2) can achieve the stated goals. Indeed, suppose temporarily that $\delta w(t) \equiv 0$, $t \in \mathbb{Z}_+$, with $\delta\xi_s(t)$ entering only the stable modes of $\chi(t)$. Even if this was the case, the neutrally stable modes of $\chi(t)$ would be indirectly affected by $\delta\xi_s(t)$ via the stable ones. However, one has to take account that (5.8) is a representation for design of the real PS (5.1). Hence, not only the positional and the incremental instantaneous values of the disturbances are bounded but also their own incremental sums. As will be seen, such a property allows one to prove the conjecture that the algorithm resulting from (6.2) can still work in the present case of set-point tracking.

Consider the incremental model (5.8) and a similarity transformation

$$\mathcal{T} :\ \chi(t) \to \chi_\epsilon(t), \qquad \chi_\epsilon(t) := [\,\delta s(t)'\, \delta q(t)'\, \epsilon(t-1)'\,]'$$

under which (5.8) is diagonalized relatively to δs,

$$\mathcal{T} := \begin{bmatrix} I_s & 0 & 0 \\ 0 & I_q & 0 \\ H_s(I_s - \Phi_s)^{-1} & 0 & I_m \end{bmatrix} \tag{6.3}$$

Such a transformation always exists as $1 \notin sp(\Phi_s)$. As can be checked, this leads to the next incremental model (IM$_\epsilon$) algebraically equivalent to (5.8)

$$\begin{cases} \delta s(t+1) = \Phi_s \delta s(t) + G_s \delta u(t) + \delta\xi_s(t) \\ \delta q(t+1) = \Phi_q \delta q(t) + G_q \delta u(t) \\ \epsilon(t) = \epsilon(t-1) + H_q \delta q(t) + W_s \delta u(t) + \delta n(t) \end{cases} \tag{6.4}$$

[2]Consider the system $x(t+1) = -x(t) + u(t) + c(t)$, $x(0) = x_0$, $c(t) = \overline{c} \cdot (-1)^t$, $\overline{\xi}_q > 0$, with $u(t)$ subject to (5.4). If $\overline{\Delta} < \overline{\Delta}_q := 2\overline{\xi}_q$ the regulation problem has no solution.

with $W_s := H_s(I_s - \Phi_s)^{-1}G_s$, $\delta n(t) := \hat{W}_s \delta\xi_s(t) + \delta\zeta(t) - \delta r(t)$ and $\hat{W}_s := H_s(I_s - \Phi_s)^{-1}$. Accordingly, from now on, the following notation will be used

$$\mathcal{A} := \begin{bmatrix} \Phi & 0 \\ \hat{H} & \mathrm{I}_m \end{bmatrix}, \quad \mathcal{B} := \begin{bmatrix} G \\ W_s \end{bmatrix}, \quad \mathcal{C} := \begin{bmatrix} \hat{H} & \mathrm{I}_m \end{bmatrix} \tag{6.5}$$

with $\hat{H} := \begin{bmatrix} 0 & H_q \end{bmatrix}$. Because (5.15) is linear in χ_ϵ,

$$\delta u_h(\chi_\epsilon) = \delta u_h^s(\delta s) + \delta u_h^q(\delta q) + \delta u_h^\epsilon(\epsilon^-) \tag{6.6}$$

where ϵ^- stands for $\epsilon(t-1)$. Thus, by (6.4) one has

$$\delta u_h^s(k|\delta s) := -\Psi_u^{-1}\mathcal{B}'(\mathcal{A}^{h-k-1})'\mathscr{G}_h^{-1}\delta\hat{s}$$

where $\delta\hat{s} := [(\Phi_s^h \delta s)' \, 0_q' \, 0_\epsilon']'$, and similarly for $\delta u_h^q(\delta q)$. Moreover

$$\delta u_h^\epsilon(k|\epsilon^-) := -\Psi_u^{-1}\mathcal{B}'(\mathcal{A}^{h-k-1})'\mathscr{G}_h^{-1}\hat{\mathcal{A}}^h\chi_\epsilon$$

where $\hat{\mathcal{A}}^h := \begin{bmatrix} 0_s & 0 & 0 \\ 0 & 0_q & 0 \\ 0 & H_q \sum_{i=0}^{h-1} \Phi_q^i & I_m \end{bmatrix}$. Eq. (6.6) allows one to consider separately the contribution in $\delta u_h(\chi_\epsilon)$ given by the disturbances $\delta\xi_s$ injected in the stable modes and the contribution caused by the disturbances δn affecting the critically unstable ones.

6.3.2 Main result

As already pointed out, (6.4) has not the structure of a control-saturated LTI system for which it makes sense to consider stability and boundedness under arbitrary l_∞-disturbances. However, not only the positional and incremental disturbances are bounded, but also any partial sum of the incremental ones, *viz.*

$$\begin{cases} |\xi_s(t)| \leq \overline{\Xi} \;\; \Rightarrow \;\; |\sum_{t=0}^{v} \delta\xi_s(t)| \;\leq\; 2\,\overline{\Xi} \\ \\ |n(t)| \leq \overline{\mathfrak{N}} \;\; \Rightarrow \;\; |\sum_{t=0}^{v} \delta n(t)| \;\leq\; 2\,\overline{\mathfrak{N}} \end{cases} \tag{6.7}$$

hold for every t, $v \in [0,\infty)$. These properties allow one prove the conjecture that, for $h(t)$ sufficiently large, there exists an interval, comprising L consecutive steps, such that the contribution to δu of any sequence $\{\delta n(t)\}_{t=0}^{l}$, $l \in \underline{L}$, which enters the integrator modes, might be of the same order of $\sum_{t=0}^{l} \delta n(t)$. Specifically, the mentioned conjecture relies on the following argument. Let $p := [\delta q' \;\; \epsilon'^-]'$ denote the neutrally stable substate of χ_ϵ, and assume that the control horizon $h(\cdot)$ grows unbounded. This implies that $\chi(\cdot)$ is unbounded. As Φ_s is a stability matrix, and $\delta u(t)$ and $\delta\xi_s(t)$ are bounded, there must be a time t large enough at which

$\|\chi_\epsilon(t)\|^2 = \|\delta s(t)\|^2 + \|p(t)\|^2 \simeq \|p(t)\|^2$. As $\sum_{j=0}^{t} \delta n(j)$ is bounded, h is chosen after such a large t, according to the restricted system with state $p(t+l) = \hat{p}(t+l) + \tilde{p}(t+l) \cong \hat{p}(t+l)$, where $\hat{p}(t+l)$ is related to the noiseless system while $\tilde{p}(t+l)$ is the response to the bounded term $\sum_{j=t}^{t+l-1} \delta n(j)$. Under these circumstances, at times $t+l$ subsequent to such a large t, $h(t+l) = h(t) - l$ until $\|\hat{p}(t+l)\|$ decreases so as to make $\|\delta s(t+l)\|$ and/or $\|\tilde{p}(t+l)\|$ comparable with $\|\hat{p}(t+l)\|$ and, hence, significant again for the selection of the horizon. This means that a "*horizon resetting mechanism*" is inherently enforced. Consequently, the conjecture that will be proved to hold in the remaining part of this chapter, is that such a mechanism prevents $h(t)$ (and the plant state) from growing unbounded.

In order to prove the mentioned horizon resetting property, it is convenient to introduce the following lemma which is fundamental for the subsequent developments.

Lemma 6.3.1 – Let $\chi := \iota + \vartheta$, $\iota := [0'_{n_x}\ \epsilon']'$, $\vartheta := [\delta x'\ 0'_\epsilon]'$, with $\mathscr{F}_h$ as in (5.16). Then, for every $j \geq 1$, one has

$$\mathscr{F}_{h+j}(k+j)\iota = \mathscr{F}_h(k)\iota + \textstyle\sum_{i=1}^{j} \mathcal{S}_i \iota \tag{6.8}$$

$$\mathcal{S}_i := \mathscr{F}_{h+i-1}(k+i-1)\mathcal{B}\mathscr{F}_{h+i}(0), \quad i \in [1, j] \tag{6.9}$$

Proof. According to Bellman's principle of optimality [LS95], an optimal trajectory remains such from each intermediate time onward for the cost-to-go. Hence, $\mathscr{F}_h(k+1) = \mathscr{F}_{h-1}(k)\mathcal{A}_h$, where $\mathcal{A}_h = \mathcal{A} + B\mathscr{F}_h(0)$. Consequently, one can write $\forall j \geq 1$

$$\begin{aligned} \mathscr{F}_{h+j}(k+j)\iota &= \mathscr{F}_{h+j-1}(k+j-1)\mathcal{A}_{h+j}\iota = \\ &= \mathscr{F}_{h+j-1}(k+j-1)(\mathcal{A} + \mathcal{B}\mathscr{F}_{h+j}(0))\iota = \\ &= \mathscr{F}_{h+j-1}(k+j-1)\iota + \mathscr{F}_{h+j-1}(k+j-1)\mathcal{B}\mathscr{F}_{h+j}(0))\iota \end{aligned}$$

where the last equality holds because, as can be checked, $\mathcal{A}\iota = \iota$. Defining

$$\mathcal{S}_i := \mathscr{F}_{h+i-1}(k+i-1)\mathcal{B}\mathscr{F}_{h+i}(0) \tag{6.10}$$

by arguing in a similar way, the proof follows by mathematical induction. □

Remark 6.3.1 – As can be seen, the proof of the Lemma 6.3.1 hinges upon the key property that $\mathcal{A}\iota = \iota$. This property clarifies, a posteriori, that arbitrary time-varying disturbances in the PS system (5.1) cannot be directly injected into the critically unstable modes. Indeed, let $\xi_q(t)$ denote such a disturbance and let $w(t) := [0'_s\ \xi'_q(t)\ 0'_\epsilon]$. Under these circumstances $\mathcal{A}w = [0'\ (\Phi_q w)'\ (H_q w)']'$, and hence, the property given in Lemma 6.3.1 would not hold true.

Theorem 6.3.1 *Consider the ANCBI system (5.1) satisfying (5.2) and (5.10) under the input-increment saturations (5.4). Let the control increment be given by* $\delta u(t) = F_{h(t)}\chi_\epsilon(t)$ *with* $F_{h(t)}$ *as in (5.11),* $\chi_\epsilon(t)$ *as in (6.4) and* $h(t)$ *chosen according to the HSL (6.2). Then, the resulting switched system is bounded-noise bounded-state* l_∞*-stable irrespective of the initial state* $x(0) \in \mathbb{R}^{n_x}$ *and possibly time-varying bounded signals* $\xi_s(\cdot)$*,* $\zeta(\cdot)$ *and* $r(\cdot)$ [3].

Proof. See Appendix B. □

Two concluding remarks are worth-saying. Suppose that the true plant state $x(t)$ in (5.1) is replaced by the vector $\hat{x}(t) := x(t) + [\vartheta'(t)\, 0']'$, where $\vartheta(t)$ is a bounded sensor-noise acting on the stable modes of the system. In such a case, the problem of interest is to find, based on a *partial* state-information vector, feedback controls

$$u = f(\hat{x}) \tag{6.11}$$

which ensure global asymptotic stability and offset-free tracking in the presence of constant disturbances and set-point r, as well as finite l_∞-induced gain of the disturbance-to-state map from ξ to x in (5.1)-(5.4) and (6.11). It is immediate to see that the conclusions of Theorem 6.3.1 hold true. In such a case the HSL (6.2) is based on the partial state information $\hat{\chi}_\epsilon(t) = [\hat{\delta s}'(t)\, \delta q'(t)\, \epsilon'(t-1)]'$, with $\hat{\chi}_\epsilon(t)$ as in (6.4), and where $\hat{\delta s}(t)$ is a filtered-estimate of $\delta s(t)$ based on observations $\phi(t) = [\gamma(t)\, \delta q'(t)\, \epsilon'(t-1)]'$, $\gamma(t) = E\delta s(t) + \varphi(t) \in \mathbb{R}^p$ with $\varphi(\cdot)$ a bounded noise [4].

Finally, to the best of the author's knowledge, possible time-variations of $\xi_q(\cdot)$ are restricted to

$$\|\delta\xi_q(t)\| \; < \; \mu(1-\mu)/\overline{M}$$

where μ is the hysteresis constant in (6.2) and $\overline{M}$ is as in (5.24) [MT08].

6.4 Examples

The remainder of this chapter is devoted to the presentation of examples that illustrate the effectiveness of the proposed technique.

[3]The closed loop system yields also offset-free tracking for the class of disturbances and the reference sequences which become constant in finite time.

[4]Similar conclusions can be derived relatively to Theorem 6.2.1.

6.4.1 Example # 1: Control of the roll angle of an aircraft

Consider the control of the roll angle of an aircraft [Veg94]. The discrete-time positional system (zero-order holder and $5\,ms$ sampling time) is as follows

$$\begin{aligned} x(t+1) &= \begin{bmatrix} 0.9956 & 0.0177 & 0.0004 \\ 0 & 0.7788 & 0.0354 \\ 0 & 0 & 1 \end{bmatrix} x(t) \\ &+ \begin{bmatrix} 0.0000 \\ 0.0007 \\ 0.0395 \end{bmatrix} u(t) + \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \xi_3(t) \end{bmatrix} \\ y(t) &= \begin{bmatrix} -3.9528 & 0 & 0 \end{bmatrix} x(t) + \zeta(t) \end{aligned} \tag{6.12}$$

We first consider the *Feasibility Property.* System responses to a step reference, for different values of h, are shown in Fig. 6.1. Here, the qualitative behavior of (5.39) and (5.40) as a function of h is confirmed in that, as h increases, δu_h tends to a constant, while u_h tends to a straight line connecting the initial and required final value of u.

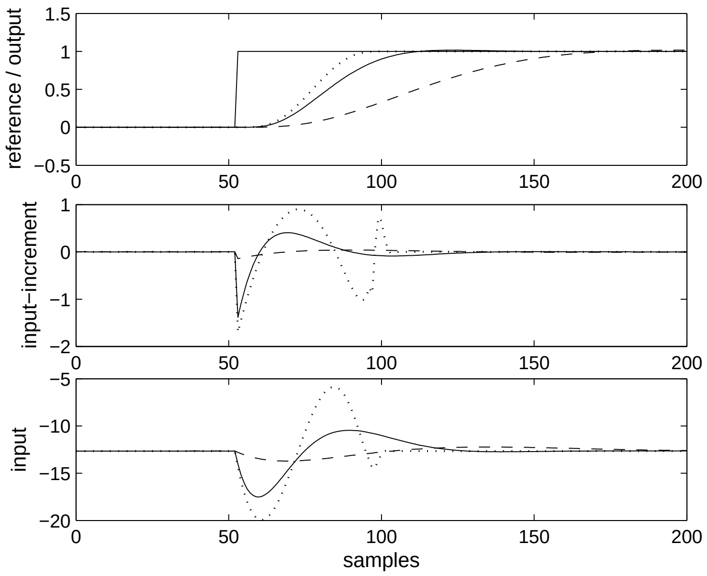

Figure 6.1: Response to step reference: Dotted line, $h = 5$; Solid line, $h = 50$; Dashed line, $h = 100$.

We next verify the horizon resetting property proved in Theorem 6.3.1 (only incremental input saturations). It can be shown that (6.12) is algebraically equivalent to systems as in (5.1) with state $x = [x_1\, x_2\, x_3]'$, where x_1 and x_2 are related to the stable modes. The aim is to stabilize (6.12) and make y to track a square wave between $[-1, 1]$ using a control action $\delta u = F_{h(t)}\chi(t)$, which saturates outside $[-5, 5]$. Accordingly, $F_h = \mathscr{F}_h(0)$, with F_h and $\mathscr{F}_h(k)$ as in (5.11), respectively, (5.18).

Simulations in Fig. 6.2 refer to disturbances uniformly distributed, $\xi_1 \in [0 \pm 0.003]$, $\xi_2 \in [1 \pm 0.05]$, $\xi_3 \in [2 \pm 10^{-5}]$ and $\zeta \in [-1 \pm 0.01]$; $\mu := 10^{-5}$ in the HSL (6.2); $h(0) = 100$. As noted, the disturbance ξ_3, entering the neutrally stable mode x_3, cannot be a time-varying arbitrary bounded sequence. In order to enforce the horizon resetting mechanism, an initial state $x(0)$ is used so that $\|s(0)\|^2 = x_1^2(0) + x_2^2(0) \ll x_3^2(0) = q^2(0)$. Fig. 6.2(c) shows, in agreement with the horizon resetting mechanism, that the control horizon decreases by one at each time-step, irrespective of the disturbances, as long as $\|s(0)\|^2 \ll q^2(0)$. Thereafter, in the presence of set-point or disturbance changes, the horizon is re-selected, at any time, in accordance with the new steady-state control.

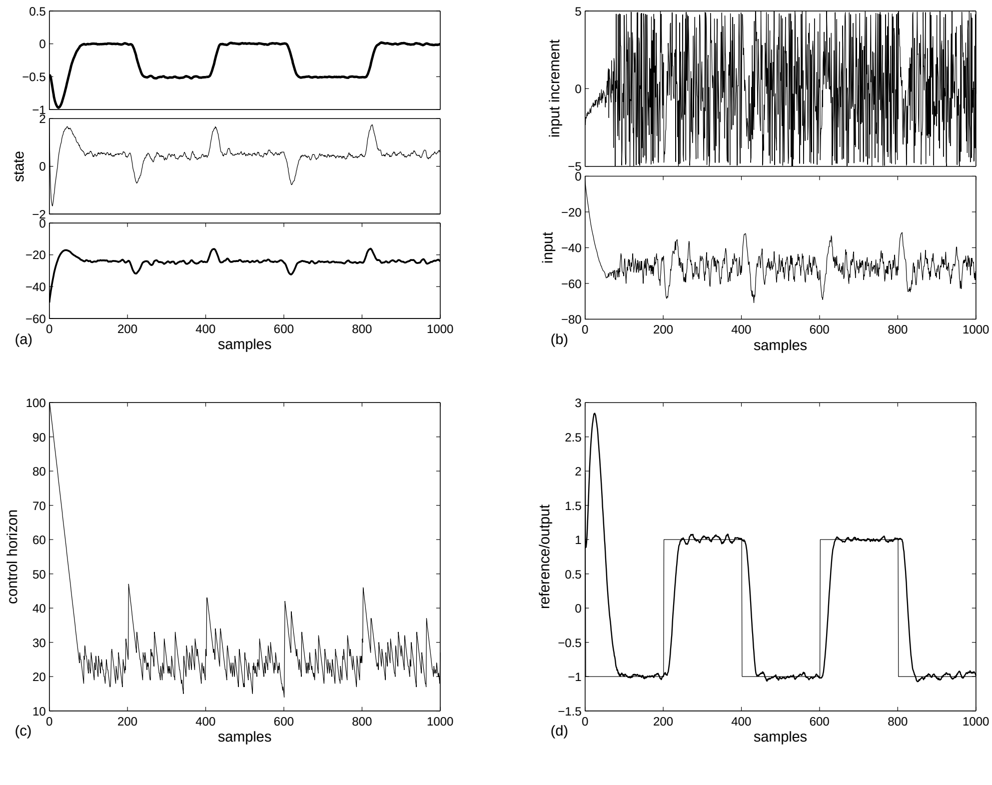

Figure 6.2: (a) State x; (b) Incremental input $\delta u \in [-5, 5]$ and input u; (c) Horizon h; (d) Square wave reference r and output y.

Of course, in the case set-point and disturbances become constant in a finite time − say t_0 − the horizon would decrease from $h(t_0)$ by one unit at each time-step, up to $\underline{h}$, with $h(t) = \underline{h}$ for every $t \geq h(t_0) - \underline{h}$.

Notice that, depending on the magnitude of the disturbances, the pair (x, u) remains in a neighborhood of the state-input $([-0.5060\,0.4038\ -24.6895]', -50.5964)$ and $([0\,0.5145\ -23.9977]', -50.5964)$ corresponding, in the steady-state with constant disturbances, to $r = 1$, respectively, $r = -1$. Suppose that the control input $u(t)$ applied to (6.12) saturate in such a way that disturbances and reference be jointly within the range of u. Simulation results indicate that, although stability results in the presence of joint positional and incremental input saturations are restricted to disturbances and reference signals which become constant in a finite time (Theorem 6.2.1), the proposed technique proves to be effective also in case of infrequent set-point changes.

A final point is worth-mentioning. In the presence of input constraints (input-increment saturations in this example), a non large enough constant horizon $\overline{h}$ makes the closed-loop unstable, even in the absence of disturbances. The control in Fig. 6.3(a) is generated by $F_{\overline{h}}\chi(t)$, $\overline{h} = 10$, when the reference is a square wave between -20 and 20, and the input increment to the plant saturates outside $[-5, 5]$. Fig. 6.3(b) shows the related divergence trend of y. In words, one cannot à priori specify the value $\overline{h}$. In this connection, in order to keep memory/computational load requirements at a moderate level, one can avail of the results of Appendix B.

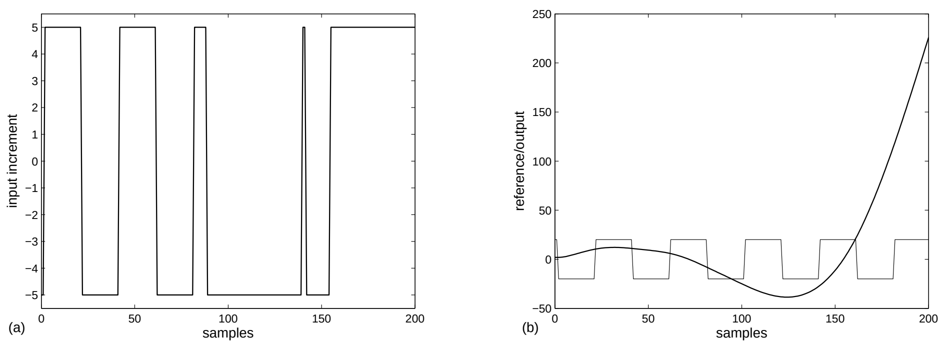

Figure 6.3: (a) Saturated input δu; (b) Divergence trend of output y with $\overline{h} = 10$.

6.4.2 Example # 2: A two carts example

Consider the same problem discussed in Section 4.3. In particular, the discrete-time positional system (zero-order holder and $0.1\,s$ sampling time) of interest is the obtained from the continuous-time plant (4.17) with $\gamma = 0.5$ [5]

[5] In the PS (6.13) it is understood that ξ enters the plant state without saturation.

$$
\begin{aligned}
x(t+1) &= \begin{bmatrix} 3.990 & -1.495 & 0.498 & -0.250 \\ 4.000 & 0 & 0 & 0 \\ 0 & 2.000 & 0 & 0 \\ 0 & 0 & 0.500 & 0 \end{bmatrix} x(t) \\
&+ \begin{bmatrix} 0.0039 \\ 0 \\ 0 \\ 0 \end{bmatrix} (\, u(t) + \xi(t)\,) \\
y(t) &= \begin{bmatrix} 0.0005 & 0.0015 & 0.0007 & 0.0001 \end{bmatrix} x(t) + \zeta(t)
\end{aligned}
\tag{6.13}
$$

where ξ and ζ correspond to n_u, and, respectively, n_y (here, for the sake of generality, a nonzero plant disturbance ξ is taken). The reference $r(t)$ is a square-wave of amplitude $\pm 2.5\ m$; Fig. 6.4 and Fig. 6.5 report simulation results relatively to the PS (6.13) with disturbances uniformly distributed, $\xi \in [0 \pm 0.003]$, and $\zeta \in [-1 \pm 0.01]$; $\mu := 10^{-5}$ in the HSL (6.2); $h(0) = 100$.

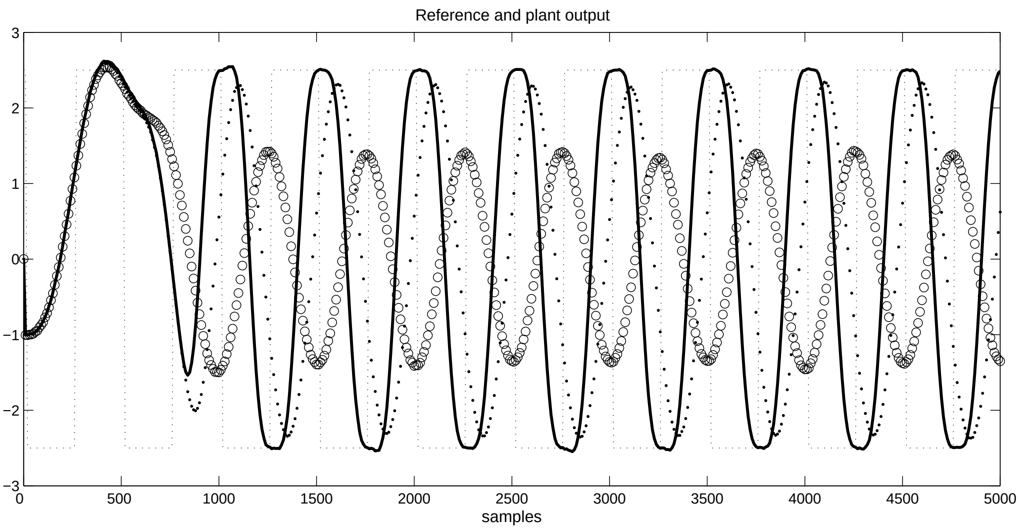

Figure 6.4: Reference and plant output for different input-increment bounds $\overline{\Delta}$: solid line, $\overline{\Delta} = 100$; $\bullet-$line, $\overline{\Delta} = 10$; $\circ-$line, $\overline{\Delta} = 1$

The same comments made in Example # 1 also applies here. In particular, as beforehand, the transiction matrix of the PS (6.13) has unitary eigenvalues. In this respect, notice that, here, the steady-state pair (x_∞, u_∞), corresponding to constant disturbances ξ, ζ and constant reference

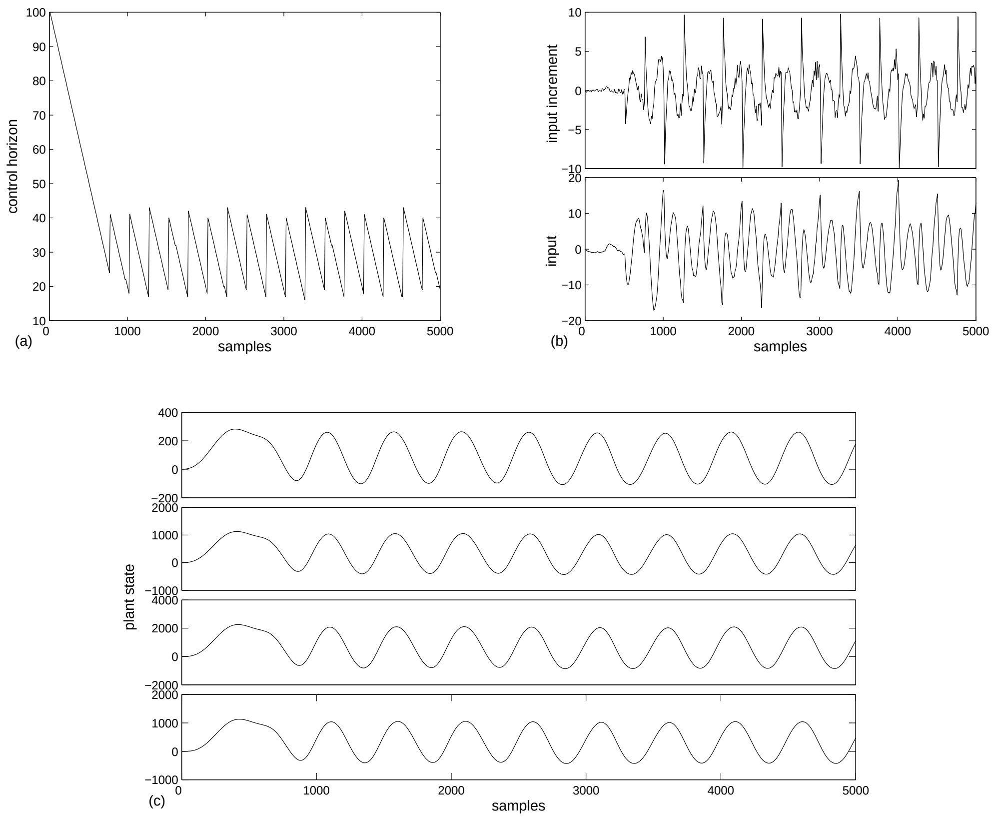

Figure 6.5: Simulation results for $\overline{\Delta} = 10$: (a) Horizon h; (b) Positional and Incremental input; (c) Plant state.

r, is such that

$$\begin{bmatrix} I_4 - \Phi & G \\ -H & 0 \end{bmatrix} \begin{bmatrix} x_\infty \\ u_\infty \end{bmatrix} = \begin{bmatrix} G\,\xi \\ \zeta - r \end{bmatrix}$$

where (Φ, G, H) is the triplet of the PS (6.13) in the form (5.1). As can be easily checked, in such a case, since Φ has unitary eigenvalues, and due to the way the disturbances enter (6.13), the pairs $(x_\infty,\, u_\infty)$'s corresponding to constant disturbances and set-points $r = 2.5$ and $r = -2.5$ are given by $(\,10^3\,[0.273,\ 1.094,\ 2.189\, 1.094]', 0\,)$ and, respectively, $(\,[-117.281,\ -469.125,\ -938.250\ -469.125]', 0\,)$. In words, here, in contrast with Example # 1, a zero steady-state control input $u_\infty = 0$ is obtained, and set-point tracking and asymptotic rejection of disturbances are taken care by the internal plant integrators which are unconstrained.

As noted beforehand, in order to avoid holding in the computer memory all feedback-gains matrices $\mathscr{F}_h$, $h \in [\,\underline{h}, \cdots, \overline{h}\,]$, one can resort to the result presented in Appendix B.

6.5 Concluding Remarks

In this chapter, attention has been devoted to predictive switching logic schemes, which provide, relatively to alternative approaches yielding comparable feasibility/performance properties, a computationally affordable solution to the set-point tracking problem of discrete-time critically unstable LTI systems subject to input saturation constraints and also affected by persistent disturbances of unknown arbitrary magnitude. In this respect, next Appendix B provides insights into Memory / Computational savings issues.

The main contribution of this chapter is the proof that all arbitrarily bounded disturbances, that can be in principle tolerated by the positional system under positional input saturations for the pure regulation problem [Mos05], are also effectively handled by the control algorithm here proposed under input-increment saturations even when the output that is used for tracking is affected by arbitrary bounded disturbances. This is a nontrivial result considering that the disturbances enter the associated incremental model in a way that, seemingly, does not conform with the canonical structure that allows to handle arbitrary bounded disturbances.

It is worth pointing out that, in the proposed scheme, robustness against persistent bounded disturbances of unknown arbitrary magnitude is acquired by switching among precomputed candidate feedback-gains stemming from the zero terminal-state minimum energy control problem [KP75, Mos95]. It is therefore of special interest to assess whether such an approach could be extended to candidate feedback-gains stemming from different underlying control problems, so as to deal with environments where both persistent disturbances and significant system model uncertainties could coexist: an issue this, as yet, unsettled and deserving research efforts in the future.

Appendix B

B.1 Memory / Computational Savings

There are two properties of the feedback-gains (5.16) which can be conveniently exploited for keeping the memory/computational requirements of the supervisory switching logic (6.2) at a moderate level.

As for *memory*, it can be shown that the virtual feedback-gains for a given horizon are computable from the ones for a larger horizon. Such a property can be called the Bellman's property as it expresses the Bellman's principle of optimality [LS95] in terms of the virtual input increments in (5.15),

$$\delta u_h(k|\chi) = \delta u_{h-1}(k-1|\mathcal{A}_h\chi), \qquad k \in \underline{h-1} \tag{B.1}$$

or, in terms of feedback-gains,

$$\mathscr{F}_{h-1}(k-1) = \mathscr{F}_h(k)\mathcal{A}_h^{-1} \tag{B.2}$$

provided that $\mathcal{A}_h$ is nonsingular. Hence, it follows from the latter equation that virtual feedback-gains for a given horizon are computable from the ones for a larger horizon.

As for *computations*, the second relevant property of the feedback-gains (5.16) can be called the Cayley-Hamilton property as it is a direct consequence of the Cayley-Hamilton theorem [Bro70]. Let $\chi_{\mathcal{A}}(z)$ be the characteristic polynomial of $\mathcal{A}$

$$\chi_{\mathcal{A}}(z) := det(zI - \mathcal{A}) = z^n + a_1 z^{n-1} + \cdots + a_{n-\rho} z^{\rho} \tag{B.3}$$

where $a_{n-\rho} \neq 0$, ρ denotes the number of the zero roots of $\chi_{\mathcal{A}}(z)$ and $n = \dim \mathcal{A}$. By the Cayley-Hamilton theorem, it follows that,

$$\chi_{\mathcal{A}}(\mathcal{A}) = \mathcal{A}^n + \cdots + a_{n-\rho}\mathcal{A}^{\rho} = 0 \tag{B.4}$$

Therefore, by the form of the feedback-gains,

$$\mathscr{F}_h(k) = c_1 \mathscr{F}_h(k-1) + \cdots + c_{n-\rho} \mathscr{F}_h(k-n+\rho) \tag{B.5}$$

for $n-\rho \leq k \leq h-1$, and

$$c_i = -a_{n-\rho-1}/a_{n-\rho}, \qquad i \in \underline{n-\rho},\ a_0 = 1 \tag{B.6}$$

As $\delta u_h(k|\chi) = \mathscr{F}_h(k)\chi$, post-multiplying each term in (B.5) by the current state χ

$$\delta u_h(k|\chi) = c_1 \delta u_h(k-1|\chi) + \cdots + c_{n-\rho} \delta u_h(k-n+\rho|\chi) \tag{B.7}$$

Eq. (B.5) and (B.7) imply that, in order to perform the admissibility test $\mathcal{M}_h(\chi) \leq 1$ for the virtual incremental and positional input sequences, it suffices to store the first $n-\rho$ feedback-gains $\mathscr{F}_h(k)$, $k+1 \in \underline{n-\rho}$, compute the first $n-\rho$ input increments $\delta u_h(k|\chi) = \mathscr{F}_h(k)\chi$, $k+1 \in \underline{n-\rho}$, while all the remaining ones in the sequence can be generated via the recursions (B.7).

B.2 Proof of Theorem 6.3.1

In order to prove Theorem 6.3.1 it is convenient to avail of the following lemma.

Lemma B [Mos05]. Consider the model (6.4) and its related $M_h(\chi_\epsilon)$ defined in (5.23). Then, there exist large enough integers h, N, $h-N > 0$, such that the two following inequalities jointly hold

$$M_h(\chi_\epsilon) \leq \gamma M_{h-N}(\chi_\epsilon) \tag{B.8}$$

$$\lambda^h \leq c/N \tag{B.9}$$

for γ, $c > 0$, $\lambda \in (0,1)$ and for every $\chi_\epsilon \in \mathbb{R}^n$. $\square$

Proof of Theorem 6.3.1.

– *Step 1: Analysis of the control law.* For the sake of notational simplicity, let

$$|\delta u_h(\chi_\epsilon)| \leq b$$

where $|\delta u_h(\chi_\epsilon)| \leq b$ stands for $|\delta u_h\,(k|\chi_\epsilon)|_i \leq b$, $i \in \underline{m}$ e $k+1 \in \underline{h}$, where $|\delta u|_i$ denotes the absolute value of the i-th component of δu. Let $\eta := \mu\overline{\Delta}$, with μ as in (6.2) and where $\overline{\Delta}$ follows from (5.4). Finally, the following shorthand notation will be used:

$$\delta\hat{\xi}(t) := [\delta\xi_s'(t)\ 0_q'\ 0_m']', \quad \delta\hat{n}(t) := [0_s'\ 0_q'\ \delta n'(t)]' \tag{B.10}$$

Notice that the latter equation, along with (6.6), implies

$$\delta u_h(\delta\hat{\xi}(t) + \delta\hat{n}(t)) = \delta u_h^s(\delta\hat{\xi}(t)) + \delta u_h^\epsilon(\delta\hat{n}(t)) \tag{B.11}$$

Moreover, according to Bellman's principle of optimality, if $\mathcal{A}_h = \mathcal{A} + \mathcal{B}\mathscr{F}_h(0)$ it follows that $\delta u_{h-1}(k|\mathcal{A}_h\chi_\epsilon) = \delta u_h(k+1|\chi_\epsilon)$. Consequently, one can write, for every $i \in [0,\infty)$,

$$\delta u_{h(t+i)}(k|\chi_\epsilon(t+i)) = \delta u_{h(t+i)+i}(k+i|\chi_\epsilon(t)) + \delta u_\xi^s(t+i) + \delta u_n^\epsilon(t+i) \tag{B.12}$$

where

$$\begin{aligned} \delta u_\xi^s(t+i) &:= \delta u_{h(t+i)+i-1}^s(k+i-1|\delta\hat{\xi}(t)) + \\ &+ \ldots + \delta u_{h(t+i)}^s(k|\delta\hat{\xi}(t+i-1)) \end{aligned} \tag{B.13}$$

denotes the contribution of the disturbance which enters the stable modes of χ_ϵ, while

$$\begin{aligned} \delta u_n^\epsilon(t+i) &:= \delta u_{h(t+i)+i-1}^\epsilon(k+i-1|\delta\hat{n}(t)) + \\ &+ \ldots + \delta u_{h(t+i)}^\epsilon(k|\delta\hat{n}(t+i-1)) \end{aligned} \tag{B.14}$$

is the contribution related to the critically unstable ones.

– *Step 2a: Horizon resetting mechanism.* The proof proceeds similarly to [Mos05]. Let, by contradiction, $h(\cdot)$ be unbounded. Then, there exists a subsequence $\{t_j\}_{j=1}^\infty$ such that $\lim_{j\to\infty} h(t_j) = \infty$ and $h(t) \le h(t_j)$, $t \le t_j$. By virtue of Lemma B, there exist large enough regulation horizons h and integers $L < h$, such that the following inequalities jointly hold

$$|\delta u_{h-l}^s(\delta\hat{\xi})| \le 2\hat{\Delta}\ \overline{\lambda}_{\Phi_s}^{h-L+1}\ \Xi \le \eta_1/L \tag{B.15}$$

$$|\delta u_{h-l}^\epsilon(\delta\hat{n})| \le 2\overline{\Delta}\ \overline{M}(h-L+1)^{-1}\mathfrak{N} \le \eta_2 \tag{B.16}$$

$\eta_1 + \eta_2 = \eta$, $\hat{\Delta} \in (0,\infty)$, $l \in \underline{L}$ and

$$M_h(\chi_\epsilon) \le \gamma M_{h-N}(\chi_\epsilon) \tag{B.17}$$

for $\gamma = 1 - 2\eta_2/\overline{\Delta} - [(L+1)/L](\eta_1/\overline{\Delta})$, $N \ge L+1$ and every $\chi_\epsilon \in \mathbb{R}^n$.

Choose an j so large that, with $\tau := t_j$, $h = h(\tau)$ satisfy (B.15), (B.16) and (B.17). Because of switching criterion (6.2),

$$|\delta u_{h(\tau)}(\chi_\epsilon(\tau))| \le \overline{\Delta} - \eta \tag{B.18}$$

as $h(\tau-1) \le h(\tau)$. So, according to Bellman's principle, if $\chi_\epsilon(\tau+l) = \mathcal{A}_{h(\tau)-l+1}\chi_\epsilon(\tau+l-1) + [\delta\xi_s'(\tau+l-1)\ 0'\ \delta n'(\tau+l-1)]'$, $l \in \underline{L}$, one has

$$|\delta u_{h(\tau)-l}(\chi_\epsilon(\tau+l))| \le \overline{\Delta} - [(L-l)/L]\eta_1 \tag{B.19}$$

provided that, with $h(\tau+l)=h(\tau)-l$, we can write for (B.14)

$$|\delta u_n^\epsilon(\tau+l)| \leq \eta_2 \tag{B.20}$$

Put in other words this means that, for $l \in \underline{L}$, the control horizon h is allowed to decrease by one unit at each time-step, provided that (B.20) holds true.

– *Step 2b: Horizon $h(t_j)$ is never exceeded.* Let $N \geq L+1$ be the smallest integer at which $M_{h(\tau)-N+1}(\chi_\epsilon(\tau+N-1))<1$ and $M_{h(\tau)-N}(\chi_\epsilon(\tau+N)) \geq 1$. By (B.17), for every $k \in \underline{h(\tau)}$ and $i \in \underline{m}$, one has

$$\begin{aligned} |\delta u_{h(\tau)}(k|\chi_\epsilon(\tau+N))|_i &\leq \\ \leq |\delta u_{h(\tau)}(k|\overline{\chi}_\epsilon)|_i + \eta_1/L + \eta_2 &\leq \\ \leq \overline{\Delta} M_{h(\tau)}(\overline{\chi}_\epsilon) + \eta_1/L + \eta_2 &\leq \\ \leq \gamma\overline{\Delta} M_{h(\tau)-N}(\overline{\chi}_\epsilon) + \eta_1/L + \eta_2 &\leq \\ \leq \gamma\overline{\Delta} + \eta_1/L + \eta_2 = \overline{\Delta} - \eta & \end{aligned} \tag{B.21}$$

with $\overline{\chi}_\epsilon := \mathcal{A}_{h(\tau)-N+1}\chi_\epsilon(\tau+N-1)$; the first inequality follows from (B.15) and (B.16), while the last one follows from $|\delta u_{h(\tau)-N}(k|\overline{\chi}_\epsilon)|_i = |\delta u_{h(\tau)-N+1}(k+1|\chi_\epsilon(\tau+N-1)|_i < \overline{\Delta}$, $k \in \underline{h(\tau)-N}$. Therefore $h(t_j+N) \leq h(t_j)$.

Let now $v := t_j+N$ and consider $|\delta u_{h(v)-1}(k|\chi_\epsilon(r+1))|_i = |\delta u_{h(v)-1}(k|\mathcal{A}_{h(v)}\chi_\epsilon(v)) + \delta u^s_{h(v)-1}(k|\delta\hat{\xi}(v)) + \delta u^\epsilon_{h(v)-1}(k|\delta\hat{n}(v))|_i$.

Recall that $|\delta u_{h(v)-1}(k|\mathcal{A}_{h(v)}\chi_\epsilon(v))|_i = |\delta u_{h(v)}(k+1|\chi_\epsilon(v))|_i \leq \overline{\Delta}-\eta$. Thus, $h(v+1) \geq h(v)$ implies $|\delta u^s_{h(v)-1}(k|\delta\hat{\xi}(v)) + \delta u^\epsilon_{h(v)-1}(k|\delta\hat{n}(v))|_i > \eta$, for some k and i. The latter, in turn, by (B.15) and (B.16), yields $h(v) \leq h(t_j)-L$. Consequently,

$$\begin{aligned} |\delta u_{h(t_j)}(k|\chi_\epsilon(v+1))|_i &= \\ = |\delta u_{h(t_j)}(k|\mathcal{A}_{h(v)}\chi_\epsilon(v)) &+ \\ +\delta u^s_{h(t_j)}(k|\delta\hat{\xi}(v)) + \delta u^\epsilon_{h(t_j)}(k|\delta\hat{n}(v))|_i &\leq \\ \leq \gamma\overline{\Delta} M_{h(t_j)-L-1}(\mathcal{A}_{h(v)}\chi_\epsilon(v)) + \eta_1/L + \eta_2 &\leq \\ \leq \gamma\overline{\Delta} M_{h(v)-1}(\mathcal{A}_{h(v)}\chi_\epsilon(v)) + \eta_1/L + \eta_2 &\leq \\ \leq \gamma(\overline{\Delta}-\eta) + \eta_1/L + \eta_2 < \overline{\Delta} - \eta & \end{aligned}$$

Hence, $h(t_j+N+1) \leq h(t_j)$. By arguing again in a similar way, one proves by mathematical induction that

$$h(t_j+k) \leq h(t_j), \quad k \in \mathbb{Z}_+ \tag{B.22}$$

provided that (B.20) holds.

– *Step 3: Fulfillment of* (B.20). To see this, rewrite (B.14) for $h(\tau+l) = h(\tau) - l, \;\; l \in \underline{L}$,

$$\begin{aligned} \delta u_n^\epsilon(\tau+l) &:= \delta u^\epsilon_{h(\tau)-1}(k+l-1|\delta\hat{n}(\tau)) + \\ &+ \;\; \ldots + \delta u^\epsilon_{h(\tau)-l}(k|\delta\hat{n}(\tau+l-1)) \end{aligned}$$

By virtue of Lemma 6.3.1, the j-th component of $\delta u_n^\epsilon(\tau+l)$ is given by

$$\begin{aligned} &\delta u_{h(\tau)-l+j}(k+j|\delta\hat{n}(\tau+l-j-1)) = \\ =\; &\mathscr{F}_{h(\tau)-l+j}(k+j)\delta\hat{n}(\tau+l-j-1) \;\; = \\ =\; &(\mathcal{S}_0+\mathcal{S}_1+\ldots\mathcal{S}_j)\delta\hat{n}(\tau+l-j-1) \end{aligned}$$

where $\mathcal{S}_0 := \mathscr{F}_{h(\tau)-l}(k)$, $\mathcal{S}_i = \mathscr{F}_{h(\tau)-l+i-1}(k+i-1)\mathcal{B}\mathscr{F}_{h(\tau)-l+i}(0)$, $i \in [1,j]$, $j \in [1,l)$. Consequently (B.14) becomes

$$\begin{aligned} \delta u_n^\epsilon(\tau+l) = \mathcal{S}_0 \sum_{j=0}^{l-1} \delta\hat{n}(\tau+j) + \\ +\mathcal{S}_1 \sum_{j=0}^{l-2} \delta\hat{n}(\tau+j) + \ldots + \mathcal{S}_{l-1}\delta\hat{n}(\tau) \end{aligned} \tag{B.23}$$

Hence, (6.7) yields

$$|\delta u_n^\epsilon(\tau+l)| \leq 2\,(|\mathcal{S}_0|+|\mathcal{S}_1|+\ldots+|\mathcal{S}_{l-1}|)\overline{\mathfrak{N}}$$

Notice that, for every $i \in [1,j]$, $\exists\, \overline{\mathcal{S}} > 0$ such that

$$\begin{aligned} |\mathcal{S}_0| &\leq \overline{\mathcal{S}}\left|O_{h(\tau)-l}\right| \\ |\mathcal{S}_i| &\leq \overline{\mathcal{S}}\left|O_{h(\tau)-l}\right|\left|O_{h(\tau)-l+1}\right| \end{aligned} \tag{B.24}$$

where O_h stands for a quantity at least of the same order of h^{-1} as $h \to \infty$. Finally, because $l \in \underline{L}$, and $L \to \mu h$ as $h \to \infty$ (see the proof of Lemma B in [Mos05] for the necessary details), one finds

$$\begin{aligned} |\delta u_n^\epsilon(\tau+l)| &\leq 2\,\overline{\mathcal{S}}\,\left|O_{h(\tau)-L}\right|\overline{\mathfrak{N}} \\ &+ \;\; 2(L-1)\overline{\mathcal{S}}\,\left|O_{h(\tau)-L}\right|\left|O_{h(\tau)-L+1}\right|\overline{\mathfrak{N}} \\ &= \;\; 2\,\overline{\mathcal{S}}\left(\left|O_{(1-\mu)h(\tau)}\right| + \frac{\mu}{1-\mu}\,\left|O_{(1-\mu)h(\tau)}\right|\right)\overline{\mathfrak{N}} \\ &= \;\; 2\,\overline{\mathcal{S}}\,\frac{1}{1-\mu}\,\left|O_{(1-\mu)h(\tau)}\right|\,\overline{\mathfrak{N}} \end{aligned} \tag{B.25}$$

Hence (B.20) holds and this completes the proof.

Conclusions

In this thesis, adaptive and predictive switching supervisory control schemes were considered, both capable of providing stability guarantees when operating in noisy environments.

In Part I of this thesis, consideration has been given to adaptive switching control schemes, and, in particular, on how to on-line infer stability of a potential feedback-loop made up by a given candidate controller interconnected in feedback with an uncertain plant, while the latter is possibly driven by a different controller. The proposed solution consists of embodying a reference-loop identification scheme in UASC: Assuming that a nominal model is associated to the candidate controller in such a way that the resulting so-called *tuned-loop* be stable, an answer is provided by sufficient conditions expressed in terms of percentage measures of discrepancy between the potential loop and the tuned one associated to the given candidate controller. In Chapter 3, a number of possible multi-model UASC systems were considered, each one equipped with a different test functional. Their features were comparatively analyzed mainly in terms of generality of applicability, stability inference and on-line implementability. It was shown that, by considering robustness against the coprime factor uncertainty on the uncertain plant relatively to the nominal model, it is possible to construct a multi-model UASC system that not only retains the desirable asymptotic properties of UASC (it only requires that Problem Feasibility be satisfied in order to yield stability), but, being intrinsically based on an inference of stability test, can moderate the chance that destabilizing controllers be switched-on, and, consequently, dramatically reduce both the magnitude and time duration of "learning" transients after start-up. In other terms, the proposed solution, while retaining UASC model-free guaranteed stability properties, also enjoys the high switching performance capability of MMASC. Chapter 4 described efficient on-line computational architectures, and showed that the positive assessment of stability robustness of UASC against the presence of possibly persistent noises/disturbances in the loop, is also enjoyed by multi-model UASC. All stability results have been derived relatively to a generic signal norm, in accordance with UASC stability analysis developed in Chapter 2. We note that a truly adaptive switching control methodology should enable the supervisor to handle also either slowly time-varying plants or infrequently abrupt plant changes. Although the analysis has been here restricted to time-invariant plants, preliminary results [BHMT09] indicate that the positive assessment of UASC stability robustness against the presence of possible time-variations of the uncertain plant is likely to be enjoyed by multi-model UASC.

In Part II of this thesis, consideration has been given to the control of input-constrained critically unstable systems. In particular, attention has been devoted to predictive switching logic schemes, which provide, relatively to alternative approaches yielding comparable feasibility/ performance properties, a computationally affordable solution to the set-point tracking problem of critically unstable LTI systems subject to input saturation constraints and also affected by persistent disturbances of unknown arbitrary magnitude. The proposed solution is realized via a supervisory switching control scheme whereby the feedback-gain selection is made in accordance with a predictive control philosophy, and each candidate feedback-gain is tuned on to a different control-horizon in a receding-horizon control sense. Iw was shown, throughout Chapter 5 and Chapter 6, that the proposed solution ensures global asymptotic stability and offset-free tracking in the presence of constant disturbances and set-point, as well as finite l_∞-induced gain of the disturbance-to-state map in the presence of time-varying disturbances and set-point, while preserving the fulfillment of control-increment saturation constraints. In addition, Appendix B showed relevant properties of the feedback-gains which can be conveniently exploited for keeping the memory/computational load requirements of the alghorithm at a moderate level. In the proposed scheme, robustness against persistent bounded disturbances of unknown arbitrary magnitude is acquired by switching among precomputed candidate feedback-gains stemming from the zero terminal-state minimum energy control problem. It is therefore of special interest to further investigate whether such an approach could be extended to candidate feedback-gains stemming from different underlying control problems, so as to deal with environments where both persistent disturbances and significant system model uncertainties could coexist.

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