Improved estimation for object localization via sensor networks

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Abstract

The paper addresses object localization via a distributed sensor network. A centralized estimation approach is undertaken along with a selective node activation strategy to ensure energy efficiency. The attention is concentrated on compensating the scarcity of information in the data (i.e., few and noisy measurements) due to selective activation by the exploitation of available constraints. To this end, a recently developed soft-constrained estimation algorithm [1] based on linear matrix inequalities is applied to the localization problem to get an improved quality of estimation. The performance of the proposed approach is evaluated on a case-study concerning a network of direction of arrival sensors.

Keywords : *sensor networks; object localization; constrained estimation; linear matrix inequalities; least-squares estimation.*

1. Introduction

Wireless sensor networks (WSN) provide nowadays an attractive solution to many civilian and military applications including for instance surveillance, localization, tracking, environmental monitoring, exploration, classification and accomplishment of several types of missions. From a research point of view, WSNs pose a number of challenging problems (e.g. deployment, routing, localization, scheduling, power control, etc.) which stay at the convergence point between communication, computing and control. For these reasons, WSNs have attracted a considerable amount of research work over the last few years [2]. A WSN is a set of a large number of low-cost and battery-powered nodes with sensing, processing and wireless communication capabilities. Since the batteries can neither be recharged nor be replaced, a key issue in order to maximize the network lifetime is to take care of energy consumption at any stage of the network's operation. Hence, energy management becomes a key factor for a successful operation whatever is the target application of the WSN.

In this paper, the focus is on the use of a WSN, with known sensors' locations, for estimating the unknown location of an object subject to an unpredictable random motion. Although the object is possibly moving, its motion cannot be satisfactorily modeled with a dynamic state equation and, hence, the problem is faced via a static estimation approach (*localization*) instead of a dynamic estimation approach (*tracking*). Each node of the WSN provides some noisy measurements of the object's position (e.g., DOA =*direction of arrival*, TOA=*time of arrival*, TDOA= *time difference of arrival* and/or RSS = *received signal strength*) provided that the object is within a known sensing range from the node location. There are essentially two ways of dealing with localization/tracking via WSNs, namely *centralized* and, respectively, *decentralized* processing.

- In *centralized processing* each active sensor node transmits raw measurements to a central node wherein the estimation takes place. For energy efficiency, this approach must be coupled with a smart selective activation strategy [3, 4, 5] in order to choose how many and which nodes to activate in the target's neighborhood so as to attain an appropriate energy-quality tradeoff.
- In *decentralized processing* the estimation is spread over all the network's nodes which, besides sensing and processing the data, communicate the local estimates to the other nodes, at a possibly much lower rate w.r.t. the measurement rate in the centralized approach.

In this paper, centralized estimation is considered for a twofold reason: (1) the nodes are not required to have processing capabilities; (2) the unpredictability and mobility of the target require frequent data communications among nodes to have a reasonable localization accuracy and this destroys the major benefit of decentral-

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ized estimation which is the communication bandwidth reduction and the consequent energy saving.

Clearly, the selective activation strategy is of paramount importance for the quality-energy performance of the centralized estimator. In particular, to increase lifetime, very few sensors should be activated. Further, sensors of a WSN are typically low-cost and, presumably, have high measurement noise. Hence, for high energy efficiency the estimation should be performed with few and noisy measurements. In this context, for better quality of estimation it becomes important to exploit the further information provided by sensors, in addition to measurements, namely that the target location must be within sensing distance from the sensor itself. To this end, we apply a constrained estimation algorithm presented in [1] to the above described object localization problem

The rest of the paper is organized as follows. Section 2 formalizes the sensor network model and the object localization problem. Section 3 describes the application of the constrained estimation algorithm [1] to object localization. In section 4, the performance of the proposed approach is evaluated on a case-study concerning a network of DOA sensors. Finally section 5 ends the paper with some concluding remarks.

2. Sensor network and estimation problem

Let us consider a network consisting of sensors which can measure the position of a target moving in a suitable neighborhood of the sensor location (*sensing region*). In this work the position of all sensors will be assumed known; in case, it is not, a self-localization procedure must be preliminarily carried out [6, 7]. Further, it is also assumed that the sensing region of each sensor is a circle with given radius (*sensing radius*). Whenever a target is detected by one or more sensors, the sensors measure the target's position and deliver their measurements to a central node responsible for the estimation. Actually, in order to improve the estimation quality, it is possible to exploit further information provided by sensors in addition to measurements. In fact, it is known that the target's position must belong to the convex region defined by the intersection of the sensing regions of all sensors detecting the target and this additional information can possibly be employed in order to improve the overall estimation accuracy.

Let $\mathbf{x}_i = (x_i, y_i)'$ be the known position vector of sensor i and $\mathbf{x} = (x, y)'$ the unknown position vector of the target. Let $\mathscr{S}$ denote the set of sensors of cardinality N, then the subset of *active* sensors that can detect the target is given by

$$\mathscr{A} = \{i \in \mathscr{S} : \ \|\mathbf{x} - \mathbf{x}_i\| \leq r_i\}$$

where r_i is the sensing radius of sensor i and $\|\cdot\|$ denotes euclidean norm. The cardinality of $\mathscr{A}$ will be denoted as $n \leq N$; it is assumed that the sensor density is such that $n > 0$. Fig. 1 illustrates an example of a target which lies in the intersection of the sensing regions of three sensors with the same sensing radius.

Figure 1. Set of active sensors

The sensors in $\mathscr{A}$ provide noisy measurements of the target position. The following linear measurement equation is assumed for sensor i:

$$\mathbf{z}_i \ = \ \mathbf{C}_i \mathbf{x} + \mathbf{v}_i \tag{2.1}$$

where: $\mathbf{z}_i$ is the noisy measurement vector; $\mathbf{C}_i$ is a known matrix; $\mathbf{x}$ is the unknown target position vector to be estimated and $\mathbf{v}_i$ is the measurement noise vector. It is assumed that $\mathbf{v}_i$ are Gaussian, with zero mean and covariances

$$E\ \left[\mathbf{v}_i \mathbf{v}_j'\right] \ = \ \delta_{ij} \mathbf{R}_i \tag{2.2}$$

where δ_{ij} is the Kronecker index and $\mathbf{R}_i$ are symmetric positive definite matrices. The n measurements provided by the active sensors can be gathered in the vector $\mathbf{z} = [\mathbf{z}_1', \mathbf{z}_2', \ldots, \mathbf{z}_n']'$ so that the measurement equations (2.1) are rewritten in matrix form as

$$\mathbf{z} = \mathbf{C}\mathbf{x} + \mathbf{v} \tag{2.3}$$

where $\mathbf{C} = [\mathbf{C}_1', \mathbf{C}_2', \ldots, \mathbf{C}_n']'$ and $\mathbf{v} = [\mathbf{v}_1', \mathbf{v}_2', \ldots, \mathbf{v}_n']'$.

The optimal *minimum mean square error* estimate of the target's position, based on the measurements provided by the n sensors, is given by

$$\hat{\mathbf{x}} = \left(\mathbf{C}'\mathbf{R}^{-1}\mathbf{C}\right)^{-1} \mathbf{C}'\mathbf{R}^{-1}\mathbf{z} \tag{2.4}$$

where $\mathbf{R} = block - diag\{\mathbf{R}_1, \mathbf{R}_2, \ldots, \mathbf{R}_n\}$ is the covariance matrix of $\mathbf{v}$. It is well known [8] that the estimator (2.4) can be interpreted as the *weighted least squares* estimator

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \left(\mathbf{z} - \mathbf{C}\mathbf{x}\right)' \mathbf{R}^{-1} \left(\mathbf{z} - \mathbf{H}\mathbf{x}\right) \tag{2.5}$$

and that the MSE of $\hat{\mathbf{x}}$ is given by:

$$\Sigma = E\left[\left(\mathbf{x} - \hat{\mathbf{x}}\right)\left(\mathbf{x} - \hat{\mathbf{x}}\right)'\right] = \left(\mathbf{C}'\mathbf{R}^{-1}\mathbf{C}\right)^{-1} \tag{2.6}$$

The estimate (2.4) does not exploit the whole information provided by the sensors, i.e. the constraints

$$\mathbf{x} \in \mathbf{X} \stackrel{\triangle}{=} \{\mathbf{x} : \|\mathbf{x} - \mathbf{x}_i\| \leq r_i, i \in \mathscr{A}\}\,. \tag{2.7}$$

Notice that the region $\mathbf{X}$ is convex, being the intersection of convex regions, but is not polyhedral, i.e. is not expressed in terms of linear inequalities. The constraints (2.7) can be exploited in the estimation process, e.g., in the following two ways [9].

- **Hard-constrained estimation:** including (2.7) as a hard constraint in the optimization (2.5), i.e.

$$\hat{\mathbf{x}}_{\mathbf{h}} = \arg\min_{\mathbf{x}\in\mathbf{X}} (\mathbf{z}-\mathbf{C}\mathbf{x})' \mathbf{R}^{-1} (\mathbf{z}-\mathbf{H}\mathbf{x}) \qquad (2.8)$$

- **Soft-constrained estimation:** introducing an extra penalization term in the cost (2.5), i.e.

$$\begin{aligned}\hat{\mathbf{x}}_s \quad = \quad & \arg\min_{\mathbf{x}} \big\{ (\mathbf{z}-\mathbf{C}\mathbf{x})' \mathbf{R}^{-1} (\mathbf{z}-\mathbf{C}\mathbf{x}) + \\ & (\overline{\mathbf{x}}-\mathbf{x})' \overline{\Sigma}^{-1} (\overline{\mathbf{x}}-\mathbf{x}) \big\}\end{aligned} \qquad (2.9)$$

  where the parameters $\overline{\mathbf{x}}$ and $\overline{\Sigma}$ are chosen, based on the constraints (2.7), so as to guarantee that the estimate $\hat{\mathbf{x}}_s$ has a MSE lower than $\hat{\mathbf{x}}$.

3. The estimation algorithm

In this work only the *soft-constrained estimation* approach is pursued and we adopt the LMI (*Linear Matrix Inequality*) technique proposed in [1] to calculate the parameters $\overline{\mathbf{x}}$ and $\overline{\Sigma}$. This choice is motivated by the fact that, in this case, the key parameters $\overline{\mathbf{x}}$ and $\overline{\Sigma}$ of this estimator do not depend on the measurement vector but only on the set $\mathscr{A}$. Therefore, once these parameters have been calculated for a given $\mathscr{A}$, one can use the same parameters until $\mathscr{A}$ does not change. Conversely, in the *hard-constrained estimation*, the constrained optimization problem must be solved again whenever a new measurement becomes available.

In order to directly apply the results in [1], the convex region $\mathbf{X}$ must be approximated with a polytope. To this end, the specific idea adopted in this work has been to approximate $\mathbf{X}$ with a polytope whose vertices are the intersection points of the circles delimiting the sensors' surveillance regions. More precisely, the following two-step algorithm can be used to calculate the above mentioned vertices.

1. For each pair of sensors $i, j \in \mathscr{A}$ determine the intersection points $\mathbf{x}$ such that $\|\mathbf{x}-\mathbf{x}_i\| = r_i$ and $\|\mathbf{x}-\mathbf{x}_j\| = r_j$.

2. Let $\mathbf{Q}$ be the set of points obtained at the previous step and let us extract from it the subset $\mathbf{P} \subset \mathbf{Q}$ of points satisfying (2.7).

It can be proved that the elements of the discrete set $\mathbf{P} = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_m\}$ obtained by the above procedure are the vertices of a convex polytope. This polytope represents an *inner approximation* of the true intersection $\mathbf{X}$ to which the design technique of [1] can be applied. Fig. 2 shows the polytope's vertices for the same example of fig. 1.

Figure 2. Polytope

Therefore, given $\mathbf{P}$ and Σ, the parameters ($\overline{\mathbf{x}}$,$\overline{\Sigma}$) of the soft-constrained estimator (2.9) can be designed by solving the following standard LMI optimization problem [10]

$$\begin{aligned}& \min_{\overline{\mathbf{x}},\,\overline{\Sigma}} \; tr\,\overline{\Sigma} \qquad \text{subject to:} \\ & \begin{bmatrix} 2\overline{\Sigma}+\Sigma & \mathbf{p}_i-\overline{\mathbf{x}} \\ \mathbf{p}_i'-\overline{\mathbf{x}}' & 1 \end{bmatrix} \geq 0 \qquad i = 1,2,\ldots,m \\ & \overline{\Sigma} \geq 0\end{aligned} \qquad (3.1)$$

where tr denotes the trace. The estimator (2.9) with parameters ($\overline{\mathbf{x}}$,$\overline{\Sigma}$) chosen as in (3.1) will be called hereafter *soft-constrained MAP estimator* as in [1] and denoted by MAP$_s$. From a geometrical point of view [1] the ellipsoid $(\mathbf{x}-\overline{\mathbf{x}})'(2\overline{\Sigma}+\Sigma)^{-1}(\mathbf{x}-\overline{\mathbf{x}}) \leq 1$ is the one with minimum sum of the squares of the axes circumscribing the polytope, i.e. containing its vertices $\mathbf{P}$. Fig. 3 shows the polytope and the circumscribing ellipsoid with reference to the same example of figs. 1 and 2.

Figure 3. Ellipsoid $(\mathbf{x}-\overline{\mathbf{x}})'(2\overline{\Sigma}+\Sigma)^{-1}(\mathbf{x}-\overline{\mathbf{x}}) \leq 1$

4. Performance evaluation

The goal is the estimation of the position of a static target using measurements from DOA (*Direction of Arrival*) sensors. In this case the measurement equation (2.1) for the ith-sensor is

$$z_i \;=\; \theta_i + \zeta_i \qquad (4.1)$$

where $\theta_i = \angle(x-x_i) + j(y-y_i)$ is the true *bearing* angle of the target w.r.t. sensor i and ζ_i is the measurement noise assumed Gaussian, with zero mean and variance σ_ζ^2. This equation is clearly nonlinear and must be linearized in order to apply the LS and MAP$_s$ estimators. From elementary geometric considerations, the target position is related to the true bearing angles by

$$y - y_i \;=\; \tan(\theta_i)\,(x-x_i)\,. \qquad (4.2)$$

Assuming that the noise term ζ_i is small and linearizing $\tan(z_i)$ in the neighborhood of $z_i = \theta_i$, one gets

$$\tan(z_i) \;\approx\; \theta_i + \frac{1}{cos^2(\theta_i)}\,\zeta_i \qquad (4.3)$$

Using the approximation (4.3) in (4.2), the following linearized model is obtained:

$$y_i - tan(z_i)x_i \;=\; y - tan(z_i)x + v_i \qquad (4.4)$$

no. of sensors	$N = 80$
measur. noise std. dev.	$\sigma_\zeta = 5^\circ$

Table 1. Sensor network characteristics

where $v_i = (x - x_i)\dfrac{1}{\cos^2(\theta_i)}\zeta_i$ is the measurement noise in the approximated model. Notice that the covariance of v_i is:

$$\sigma_i^2 = (x - x_i)^2 \frac{1}{\cos^2(\theta_i)} \sigma_\zeta^2 \qquad (4.5)$$

depends on the unknown quantities x and θ_i. This dependence can be eliminated by considering that $(x - x_i)^2 \leq r_i^2$ and $\theta_i \approx z_i$, hence it follows that

$$\sigma_i^2 \approx r_i^2 \frac{1}{\cos^2(z_i)} \sigma_\zeta^2 \qquad (4.6)$$

The model (4.4) can be cast in the form (2.3), with a monodimensional measurement vector, by taking:

$$\mathbf{z}_i = y_i - tan(z_1)x_i,\ \mathbf{C}_i = [-tan(z_i),\ 1],\ \mathbf{R}_i = \sigma_i^2 \quad (4.7)$$

This approximated model has been used to estimate the unknown target's position $\mathbf{x}$ using the following two estimators:

- LS estimator (2.4) based on measurements only;
- MAP_s estimator (2.9) based also on the knowledge of the constrained region $\mathbf{X}$.

In order to compare the two estimators we considered the MSE (Mean Squared Error) as performance index. MSE has been evaluated via Monte Carlo simulations (1000 runs) by varying both the measurement noise realization and the sensors' location within the surveillance environment. A 10×10 square has been considered as surveillance environment wherein sensors have been placed randomly with an uniform distribution. The characteristics of the sensor network are reported in table 1. The target has been placed at position $\mathbf{x} = (5, 5)'$. The simulation results are summarized in table 2 which displays the MSE of both estimators and the % improvement of MAP_s over LS for three different choices of the sensing radius, i.e. $r = 1.5$, $r = 2$ and $r = 2.5$ assumed equal for all sensors. For fixed N, increasing r means increasing the average number of sensors that detect the target ($\bar{n}$ in the table) and, hence, reduces the size of the polytope.

From table 2 it can be noticed that, if $\bar{n}$ is sufficiently large, the MAP_s estimator yields a significantly lower MSE than LS.

5. Conclusions

The paper has tackled object localization with a sensor network using a fully centralized processing approach. To take into account constraints provided by the knowledge of the sensing radius, a modified least-squares estimator with suitably designed extra penalization term has been adopted and the performance improvement of such an estimator has been assessed via a simulation case-study. Future work will concern the development of optimal strategies for selective activation of sensors providing an user tunable energy-quality tradeoff.

Acknowledgments

The authors wish to thank Dr. Alfonso Farina of SELEX-Sistemi Integrati for interesting and fruitful discussions with him concerning the subject of this paper.

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	$\bar{n}=7$			$\bar{n}=10$			$\bar{n}=15$		
Parameter	LS	MAP_s	%	LS	MAP_s	%	LS	MAP_s	%
x	0.0448	0.0443	1%	0.0034	0.0030	12%	0.0029	0.0024	16%
y	0.0471	0.0465	1%	0.0035	0.0029	16%	0.0031	0.0026	18%

Table 2. Simulation results - MSE

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