Order types of segments in floorplan partitions

Andrei Asinowski (Mathematics, Technion)

Gill Barequet (CS, Technion)

Toufik Mansour (Mathematics, University of Haifa)

Ron Y. Pinter (CS, Technion)

1. Previous work: order types of rectangles (Eyal Ackerman's Thesis, 2006).

2. Current work: order types of segments.

Problem (Lingas, Pinter, Rivest, Shamir, 1982):

Let S be a set of n points in general position inside a rectangle M. Consider all possible partitions of M into smaller rectangles such that each point of S belongs to a segment. Find the partition that minimizes the total length of the segments.

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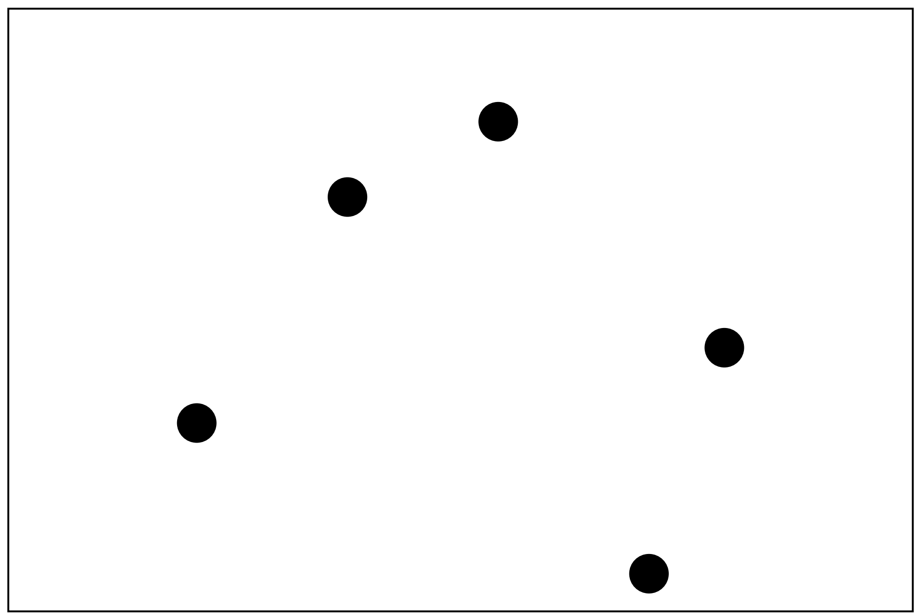

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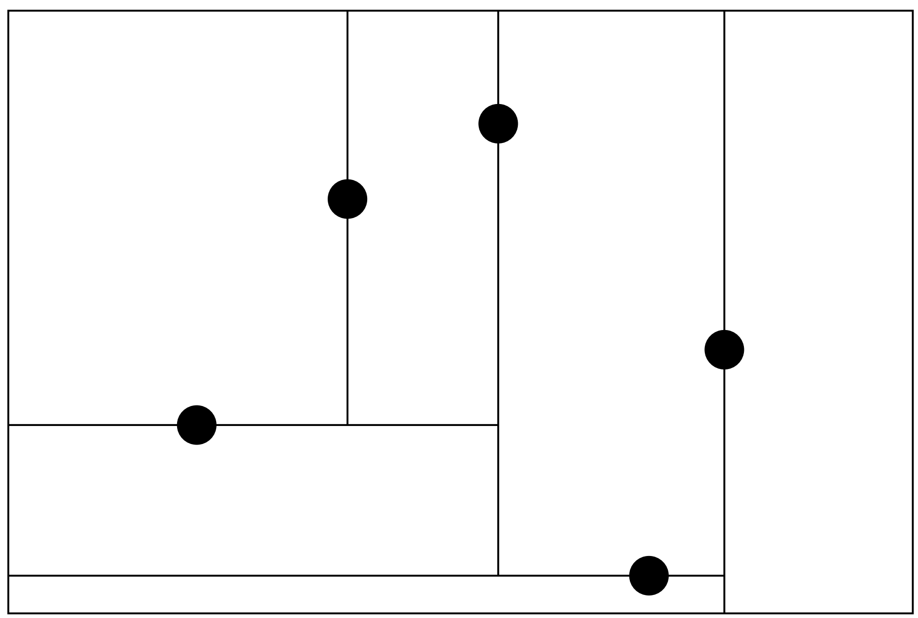

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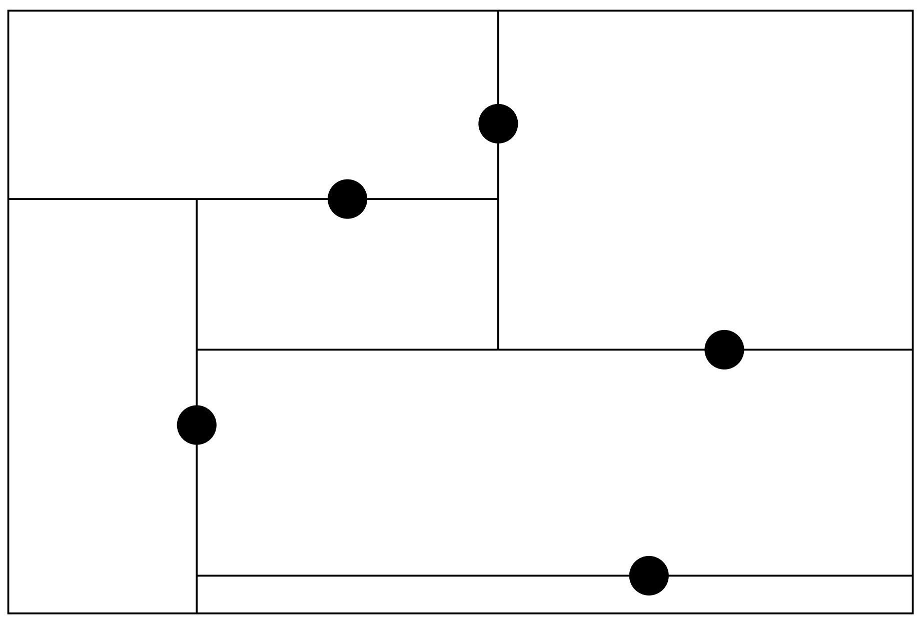
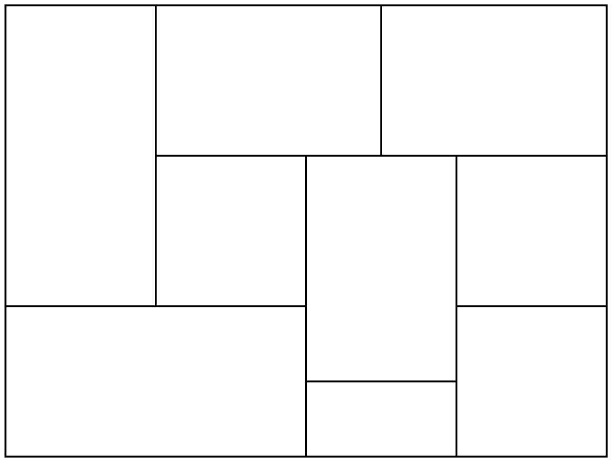


A floorplan partition:



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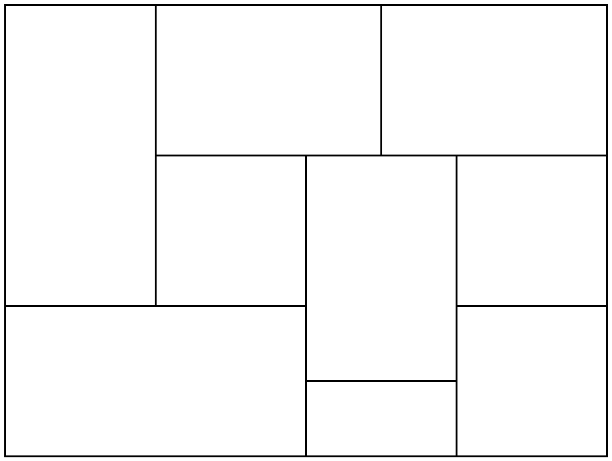
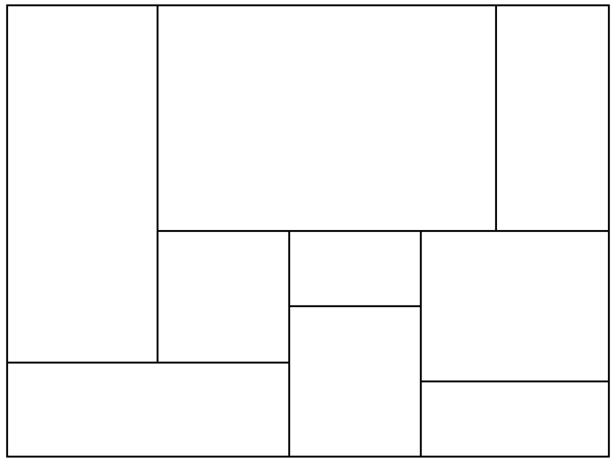
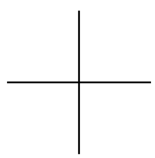




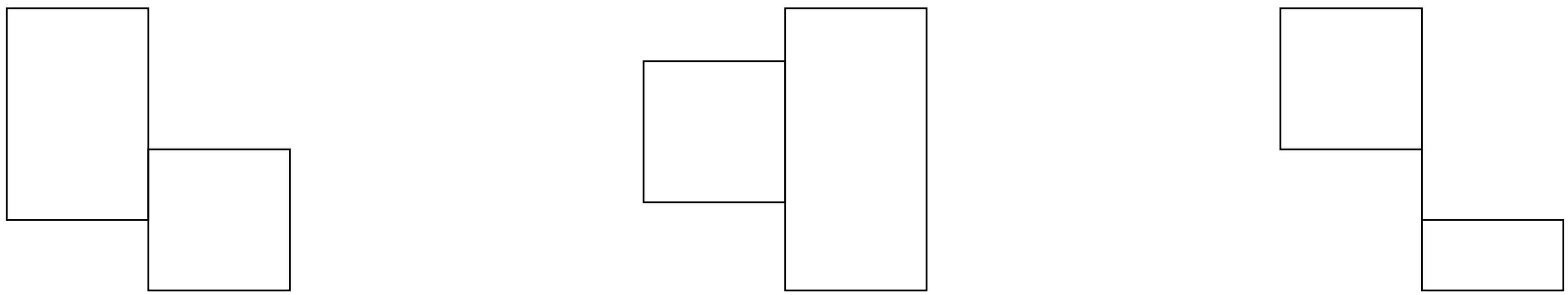
Possible junctions:

The following is not permitted:

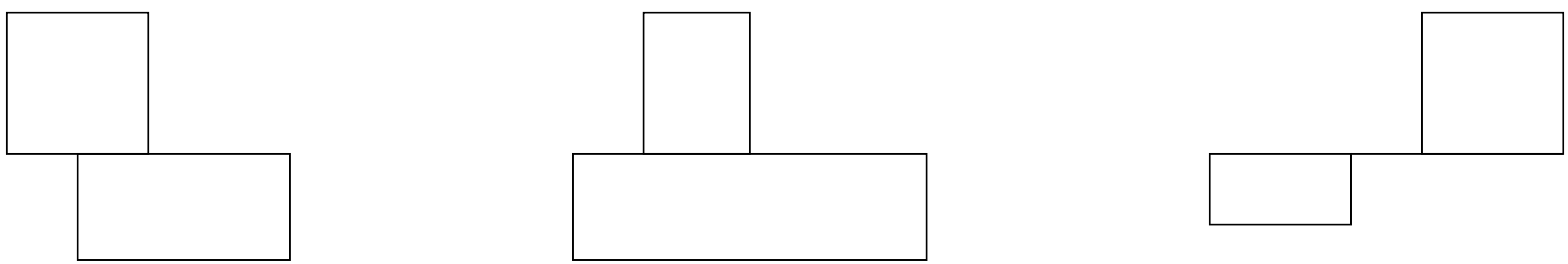


Neighborhood relationships between rectangles:

Left-right neighbors:



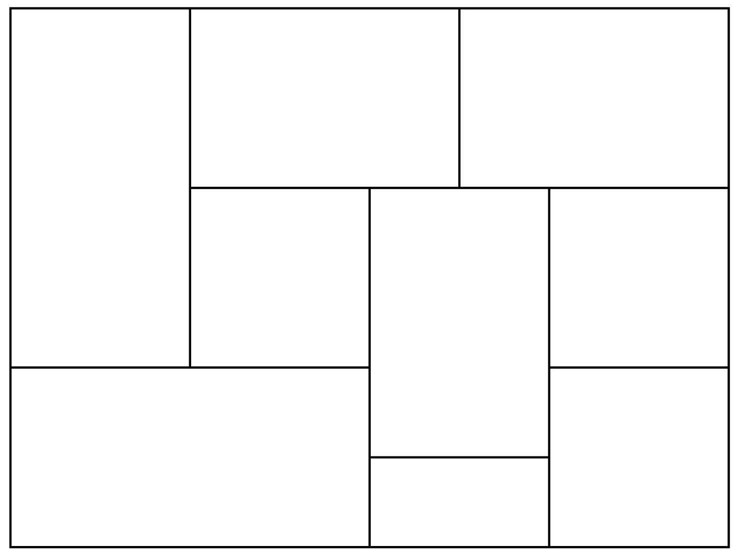
Up-down neighbors:



B is *to the right of* A:
There exist $A = A_0, A_1, A_2, \ldots, A_k = B$
so that A_{i+1} is a right neighbor of A_i.

D is *above* C:
There exist $C = C_0, C_1, C_2, \ldots, C_k = D$
so that C_{i+1} is an above neighbor of C_i.

Theorem: If A and B are two rectangles ($A \neq B$), then A and B are in precisely one neighborhood relationship.



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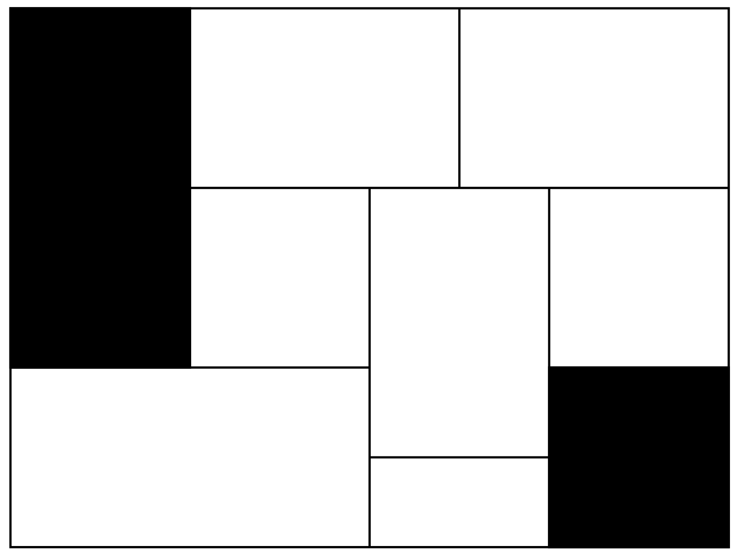

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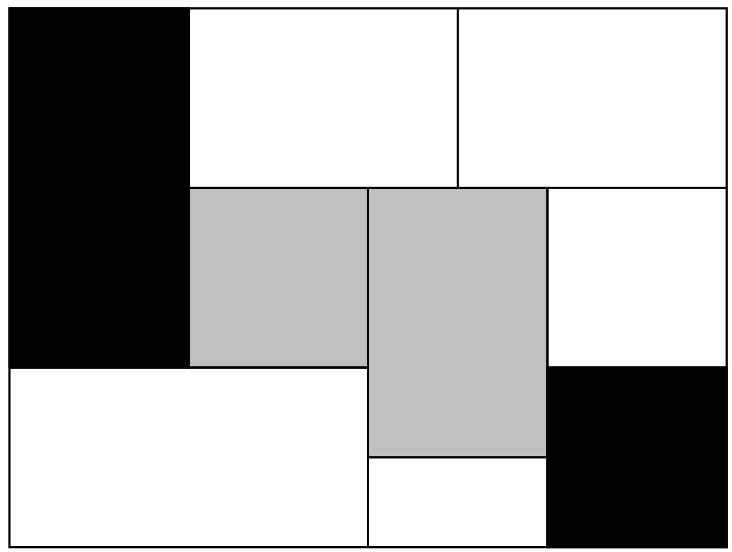

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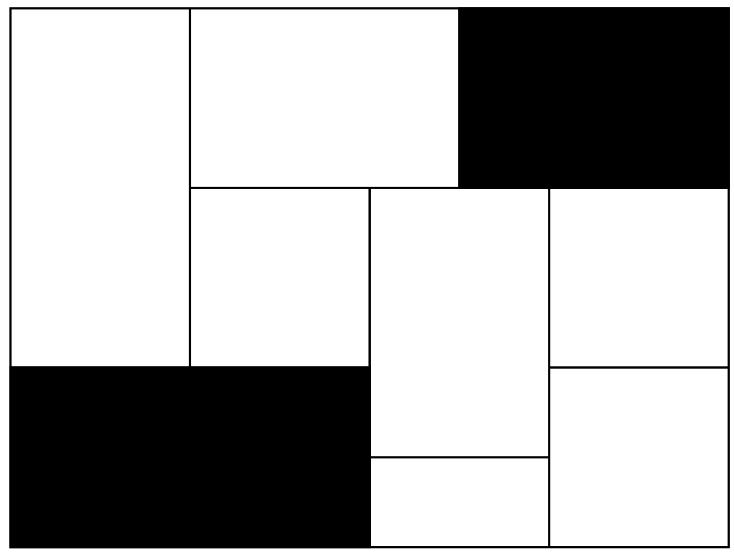

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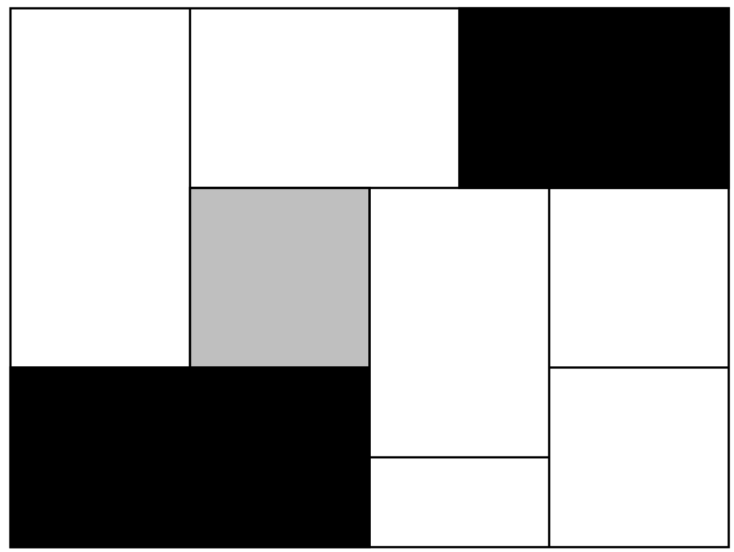
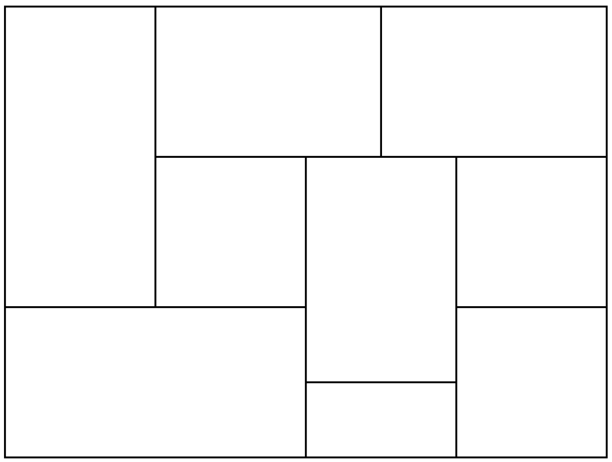
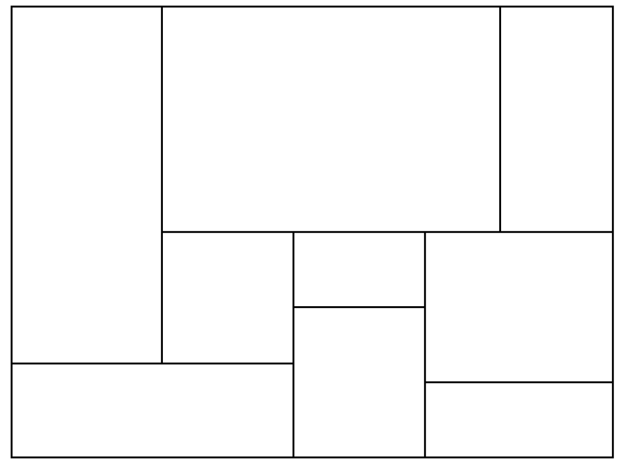




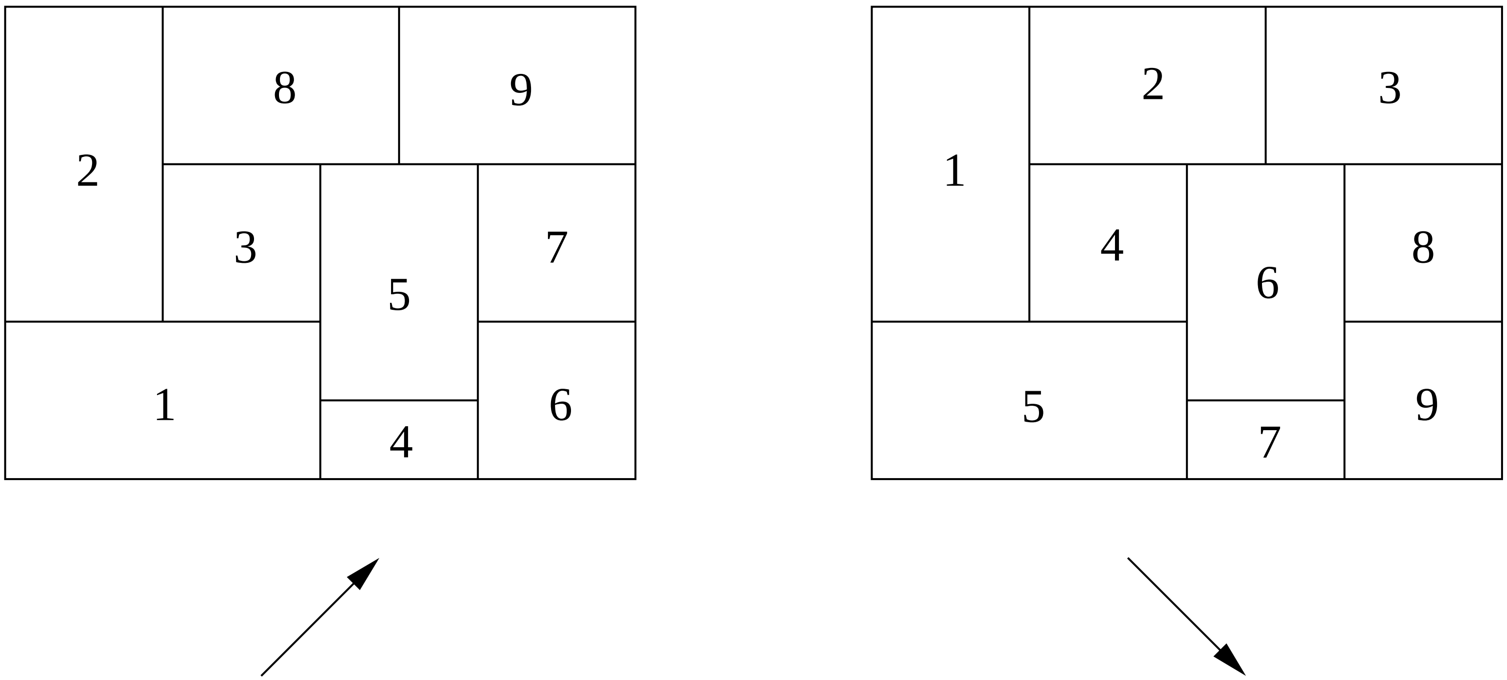


$A \nearrow B$: B is to the right of A or B is above A.

$A \searrow B$: B is to the right of A or B is below A.

These are linear orderings.

R-permutation



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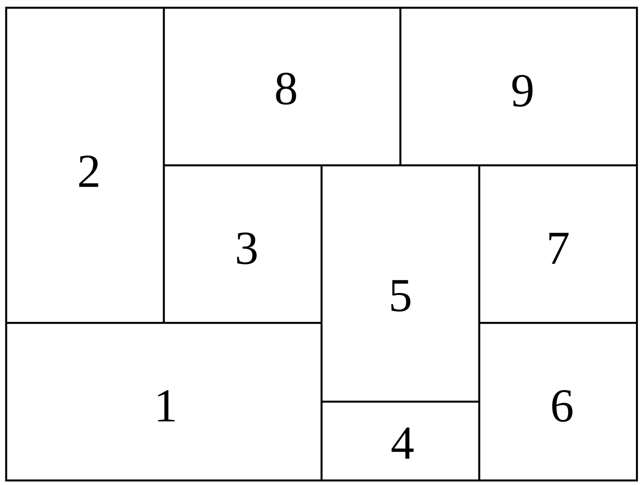
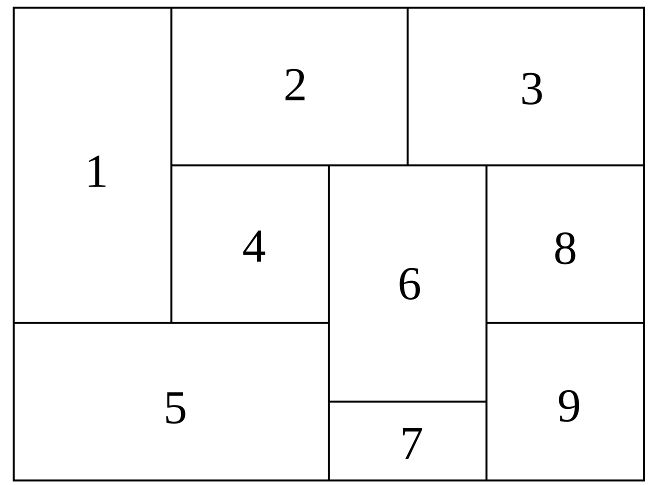
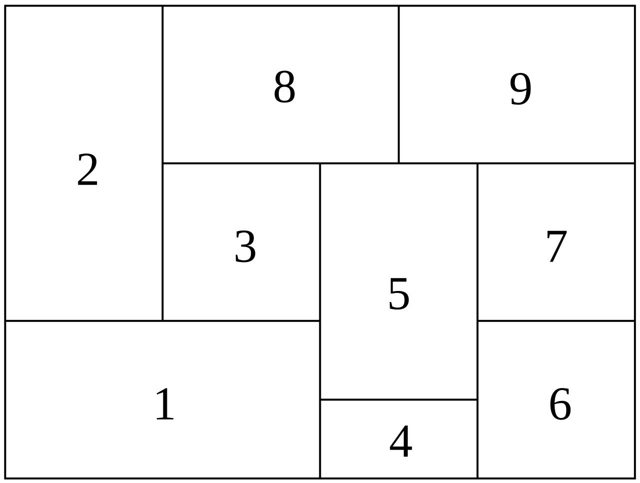
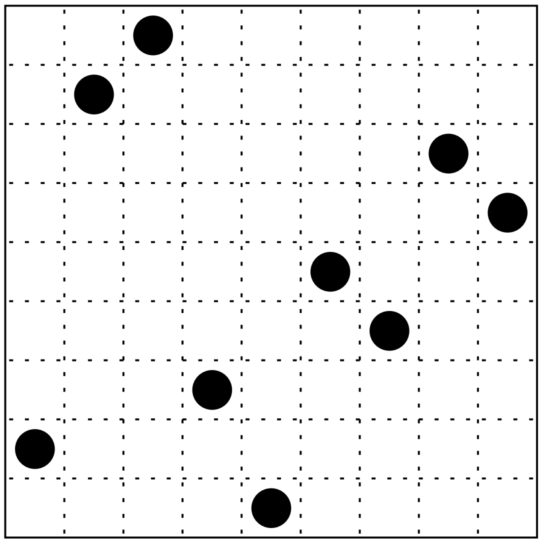




2 8 9 3 1 5 4 7 6

R-permutation

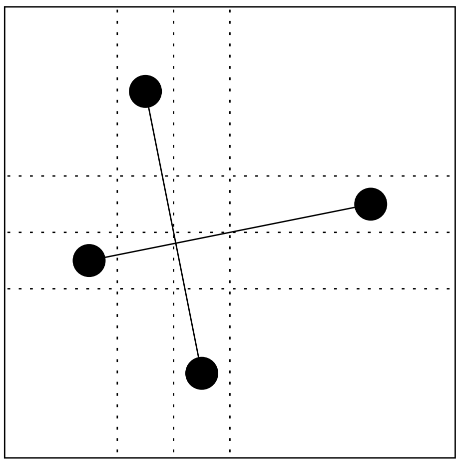
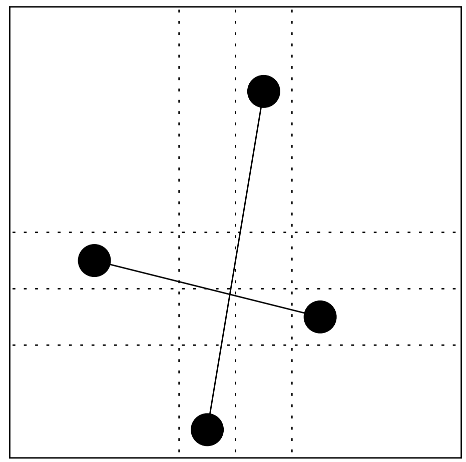




2 8 9 3 1 5 4 7 6

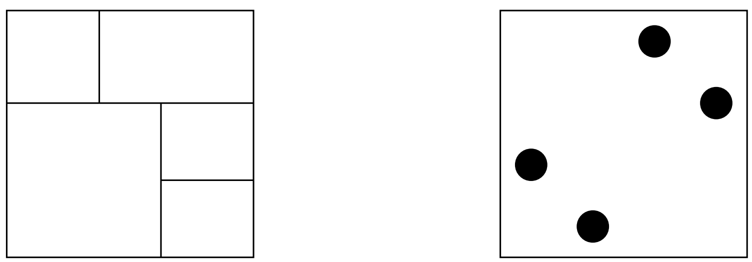
Theorem (Ackerman, Barequet, Pinter, 2006):
Permutations obtained in this way are Baxter permutations.
This correspondence is a bijection.

Baxter permutations are $(2-41-3, 3-14-2)$-avoiding
(or: $(25\bar{3}14, 41\bar{3}52)$-avoiding) permutations.

Rectangulation Problem:

Is it true that every permutation of n can be embedded in any partition with $n+1$ rectangles?



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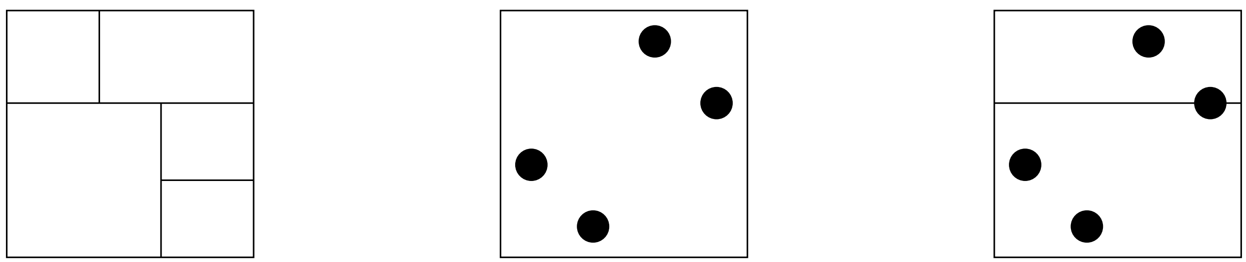

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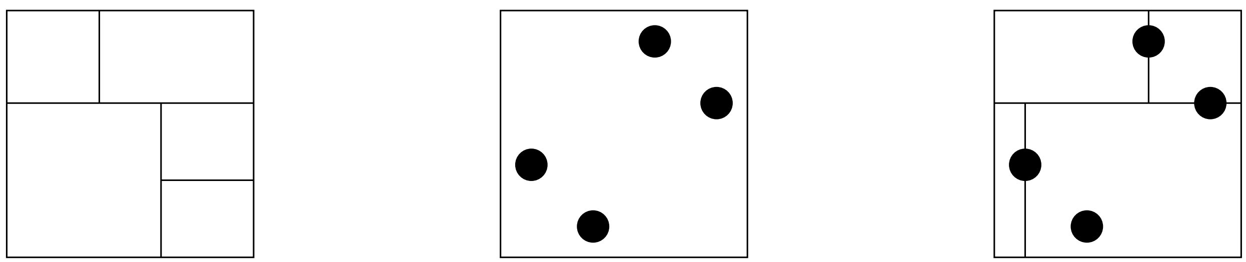

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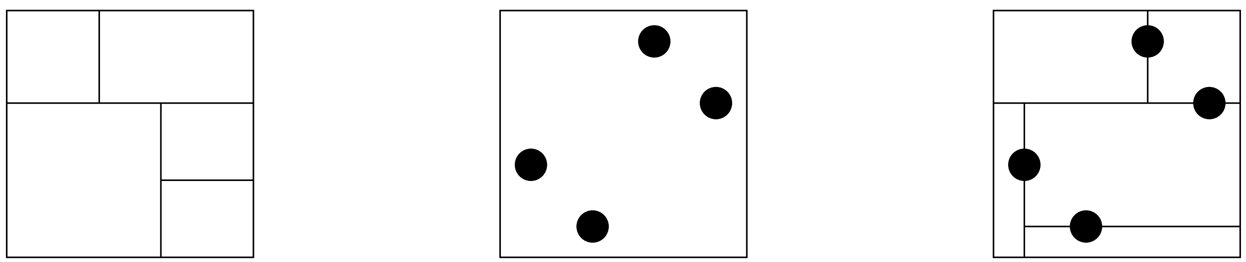
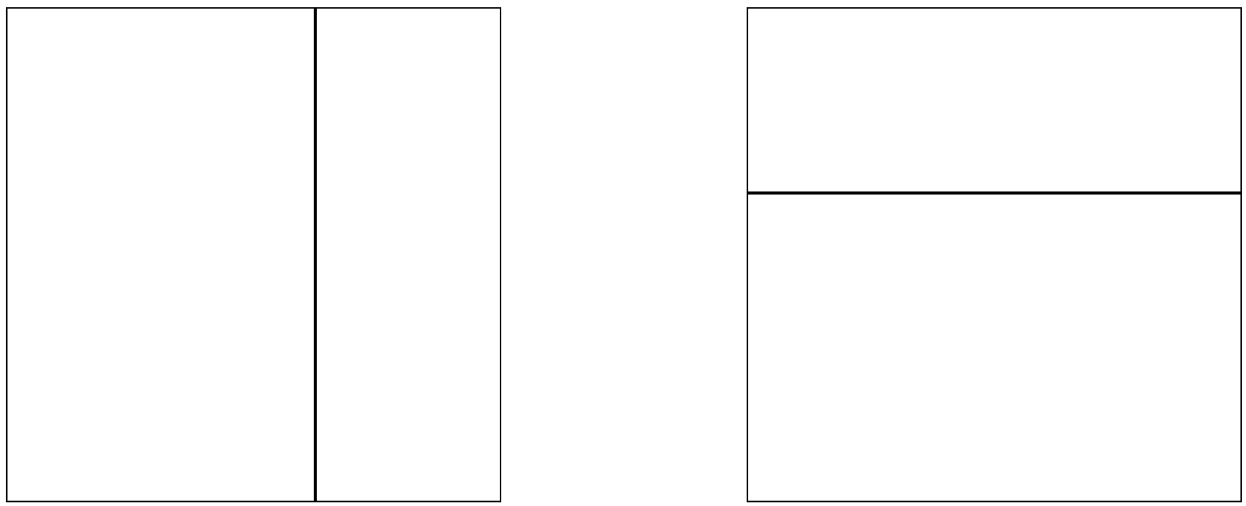


Guillotine (sliding) partitions



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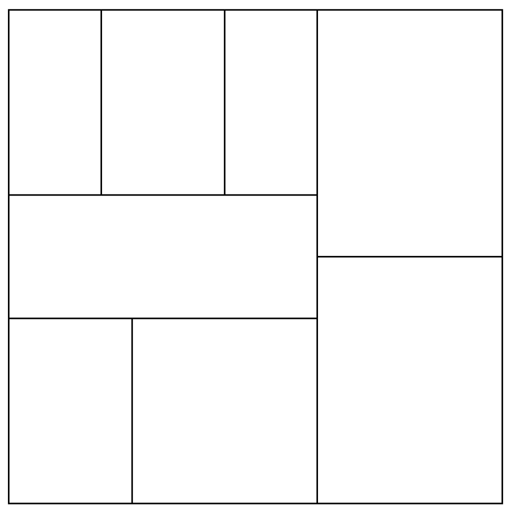
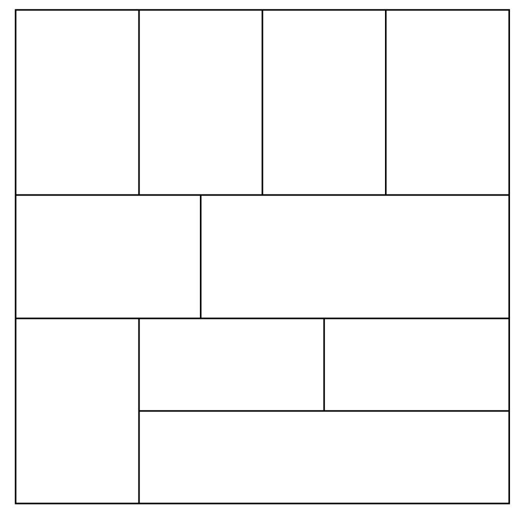
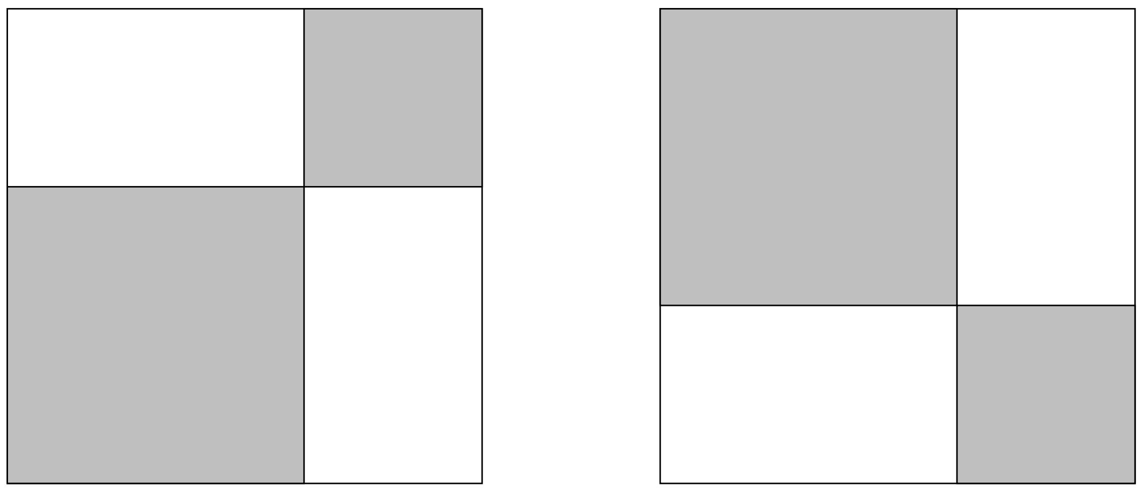




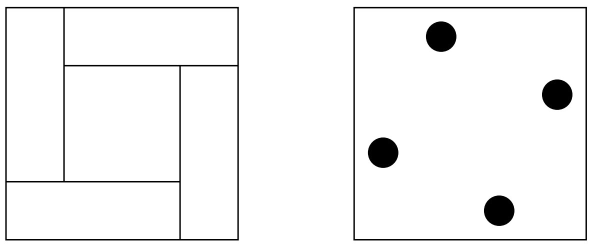
Separable permutations

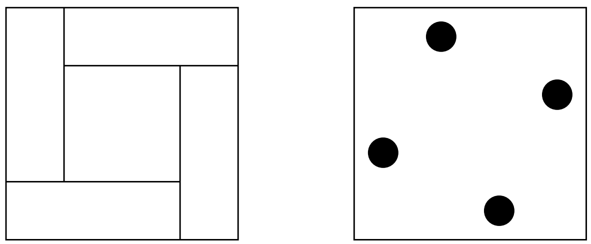


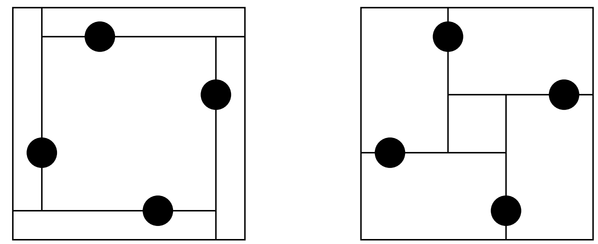
Theorem (Ackerman, Barequet, Pinter, 2006):
Let π be permutation of $[n]$,
let P be a floorplan partition with $n+1$ rectangles.
If π is separable or P is guillotine (or both),
then π can be embedded in P
(the correspondence between the points of π
and the segments of P is determined uniquely).

The weak rectangulation conjecture:
Any permutation π of $[n]$ can be embedded in any floorplan partition P with $n+1$ rectangles.

The strong rectangulation conjecture:
In addition, if π is not separable then there exists at least one floorplan partition P such that π can be embedded in P in several ways.







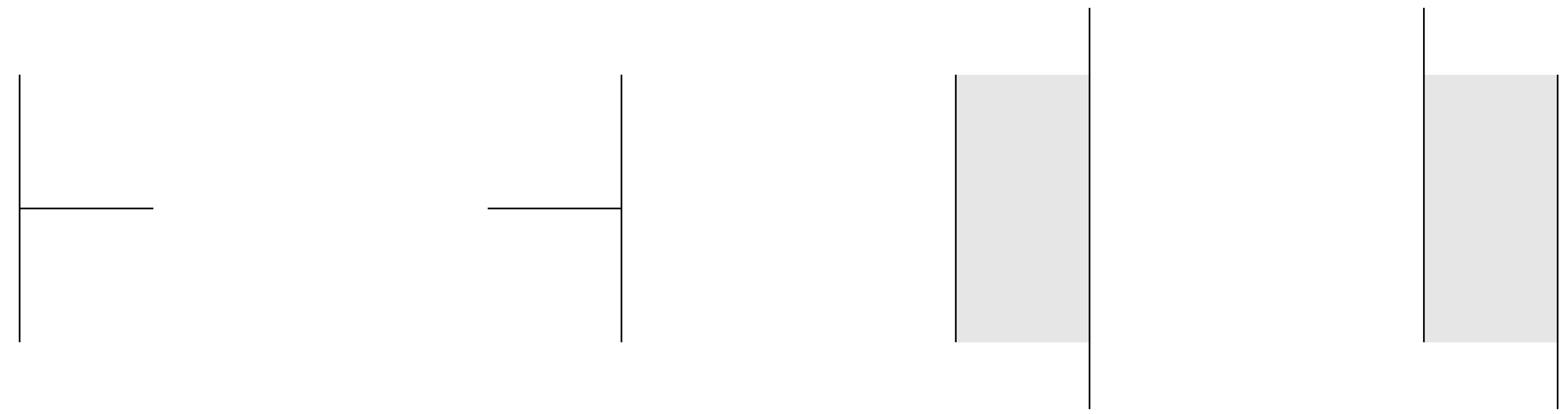
The strong rectangulation conjecture:
Let π be permutation of $[n]$, let P be a floorplan partition with $n+1$ rectangles. If π is not separable then there exists at least one floorplan partition P such that π can be embedded in P in several ways.

n	$B(n+1)$	
4	92	93
5	422	424 – 428
6	2074	2080 – 2122
7	10754	10776 – 11092
8	58202	58290 – 60524

Number of rectangulations for nonseparable permutations

Another approach: Ordering induced by **segments**

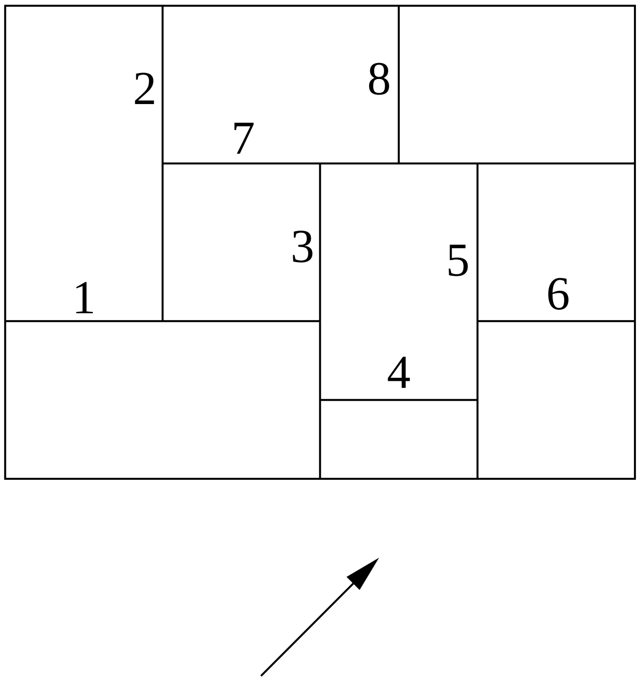
Left-right neighbors:

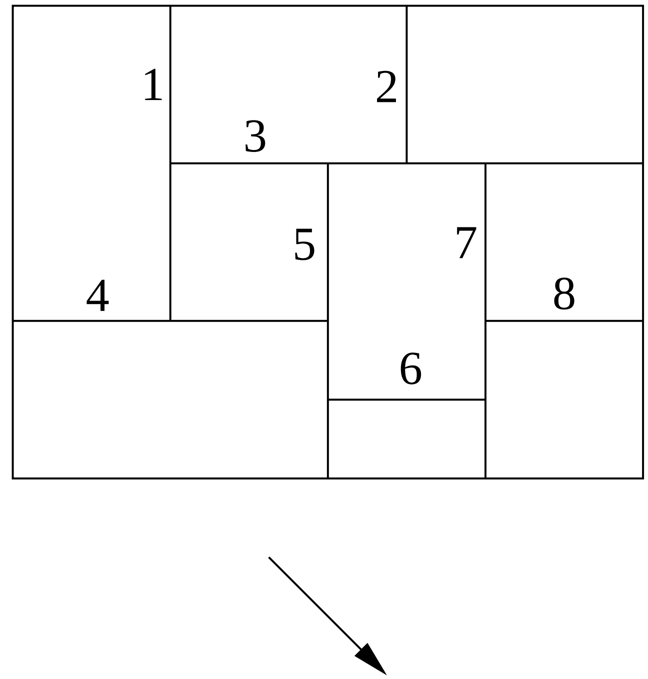


Relations $\rightarrow$, $\uparrow$, $\nearrow$, $\searrow$ are defined similarly and have similar properties:

- Each pair is comparable in exactly one of the orderings $\rightarrow$, $\uparrow$;
- The orderings $\nearrow$, $\searrow$ are linear.

S-permutation





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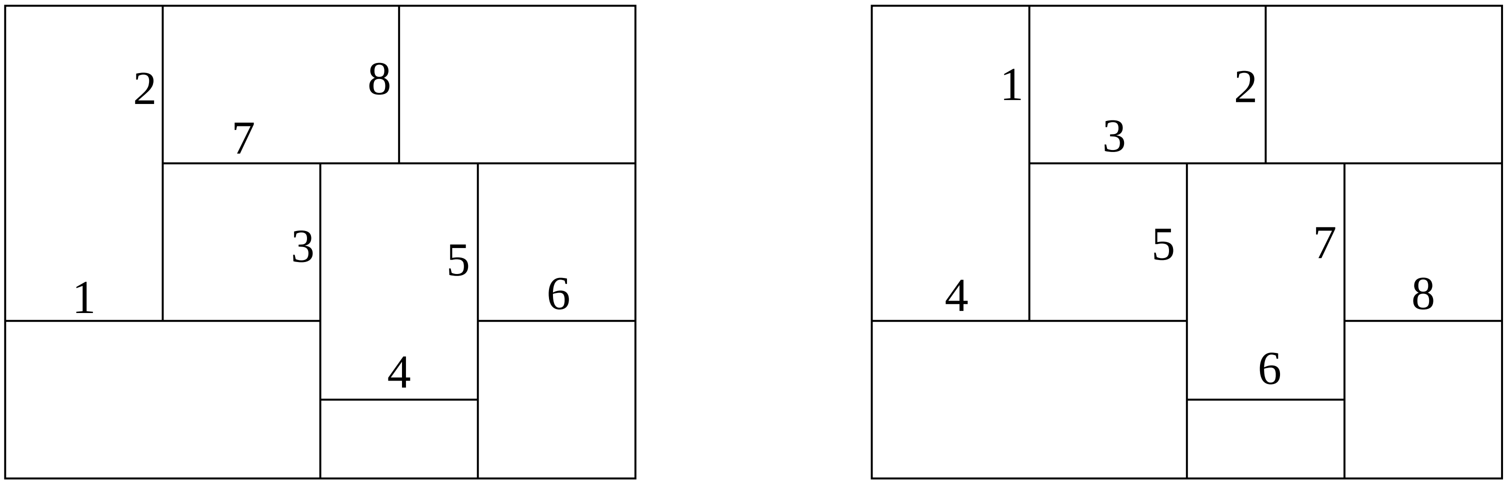
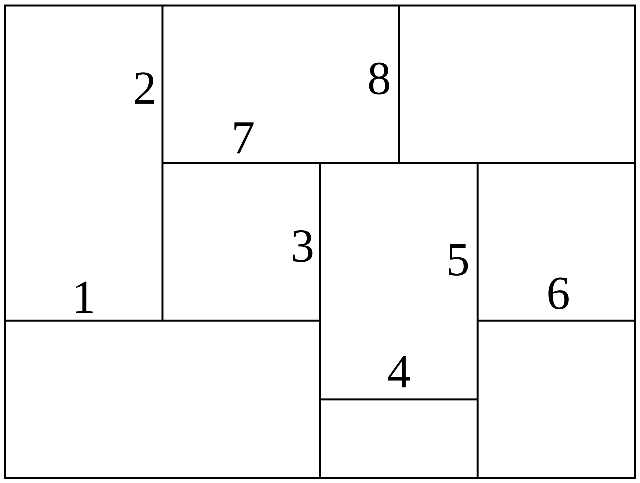
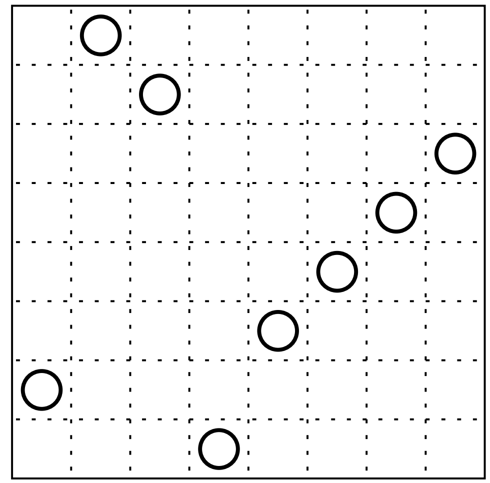


$$2\ 8\ 7\ 1\ 3\ 4\ 5\ 6$$

S-permutation





2 8 7 1 3 4 5 6

Questions:

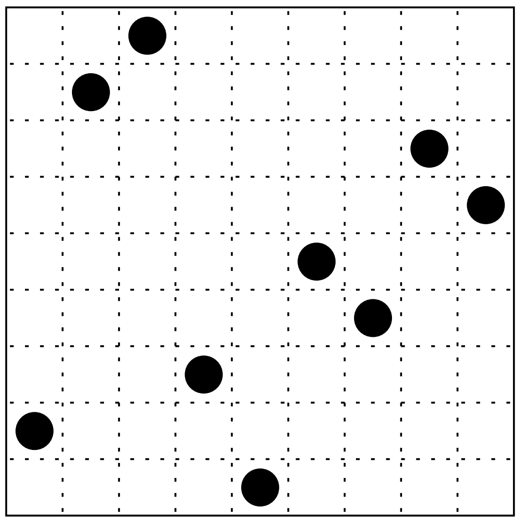
- Given a floorplan partition, how is its S-permutation related to its R-permutation?

- When do different partitions give the same S-permutation?

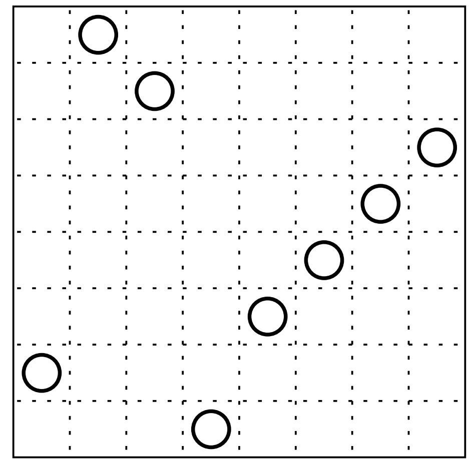
- What is the family of all possible S-permutations?

R-permutation vs. S-permutation

R-permutation vs. S-permutation

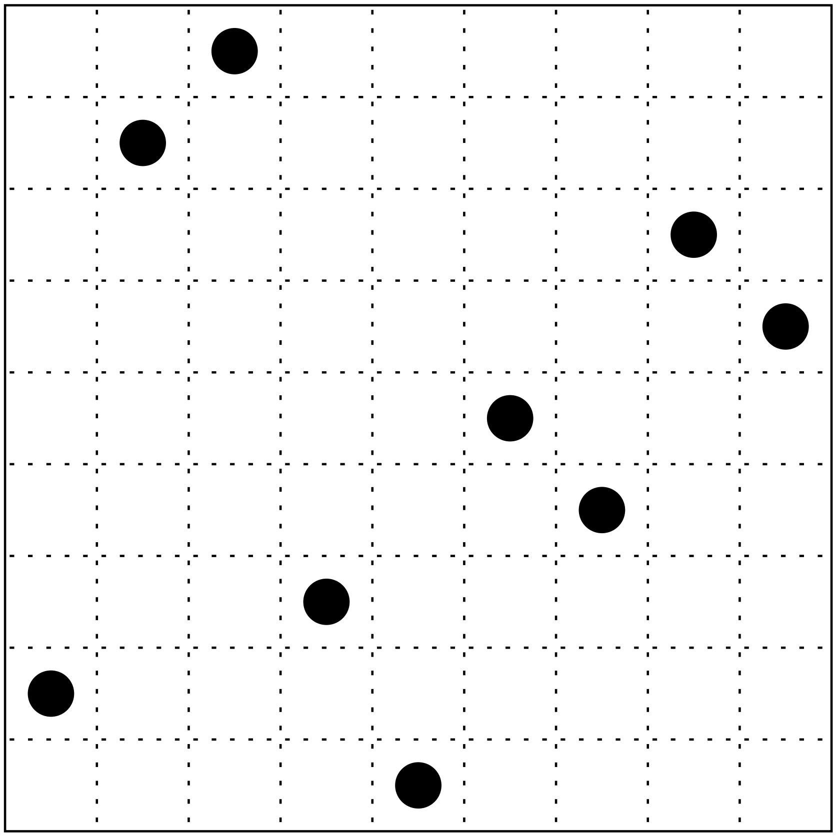


R

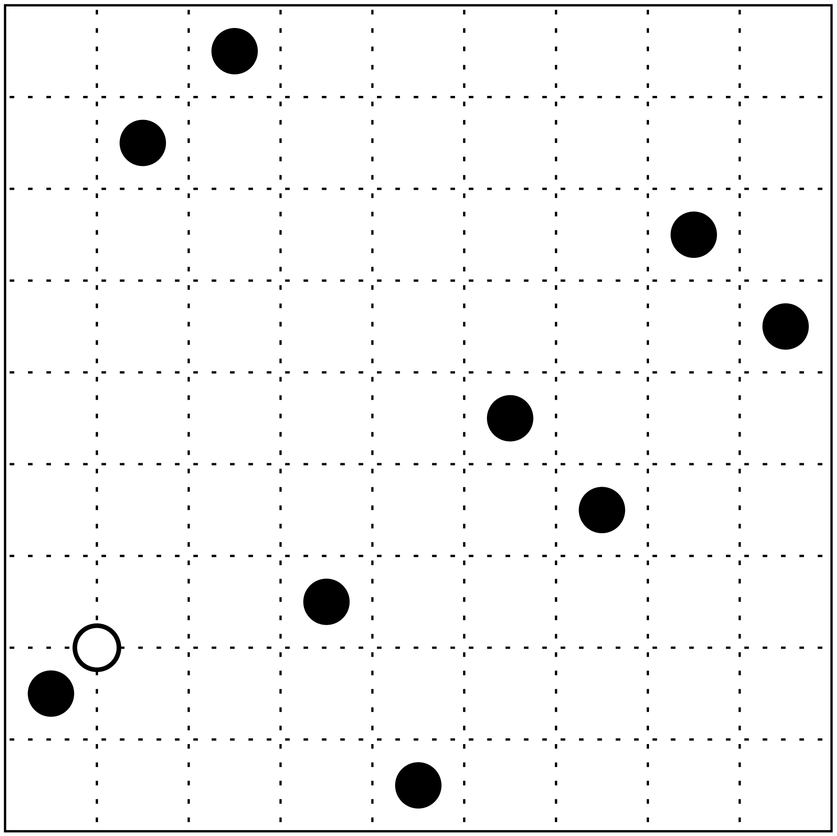


S

R-permutation vs. S-permutation



R-permutation vs. S-permutation



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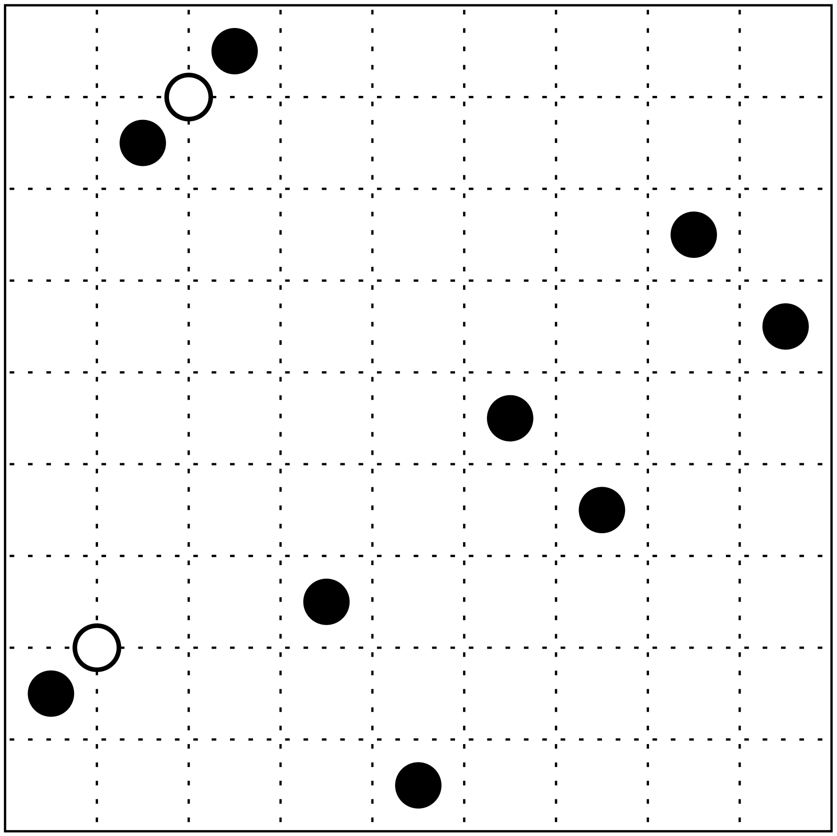

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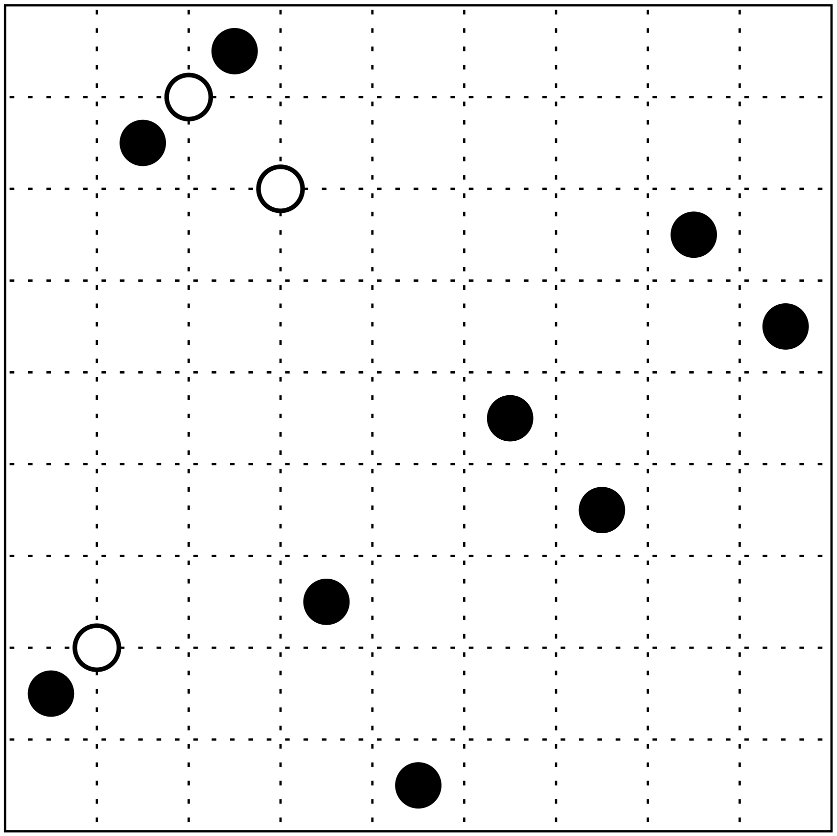

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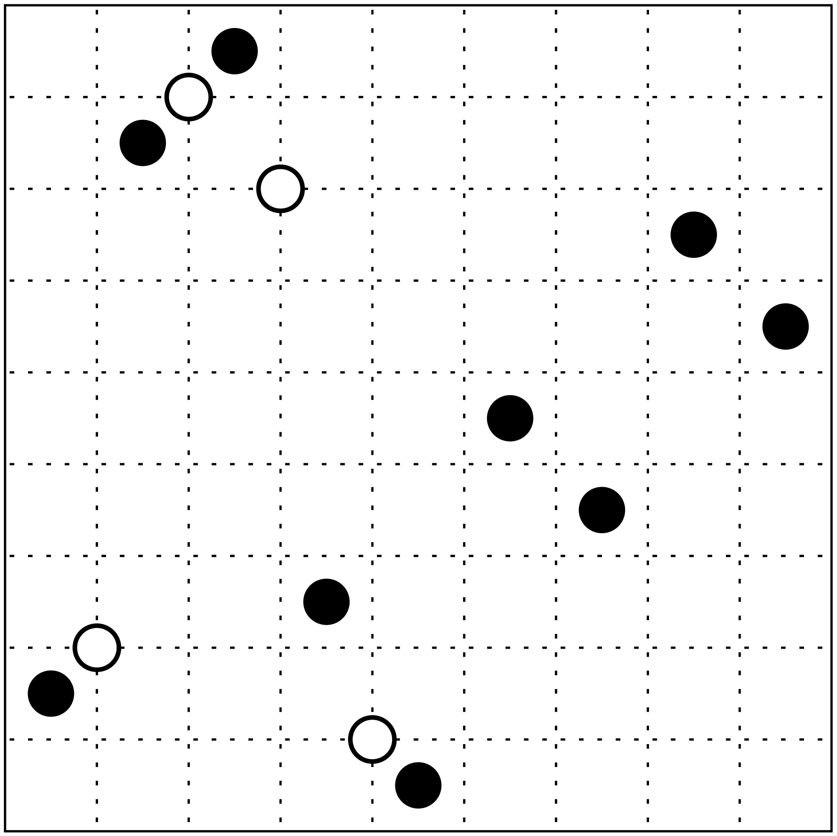

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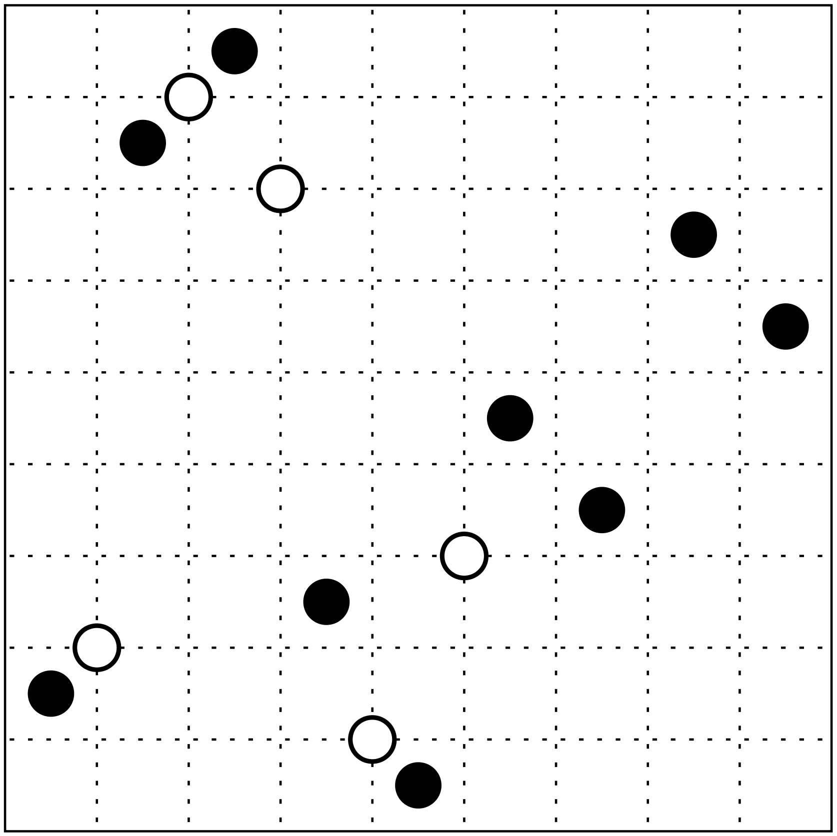

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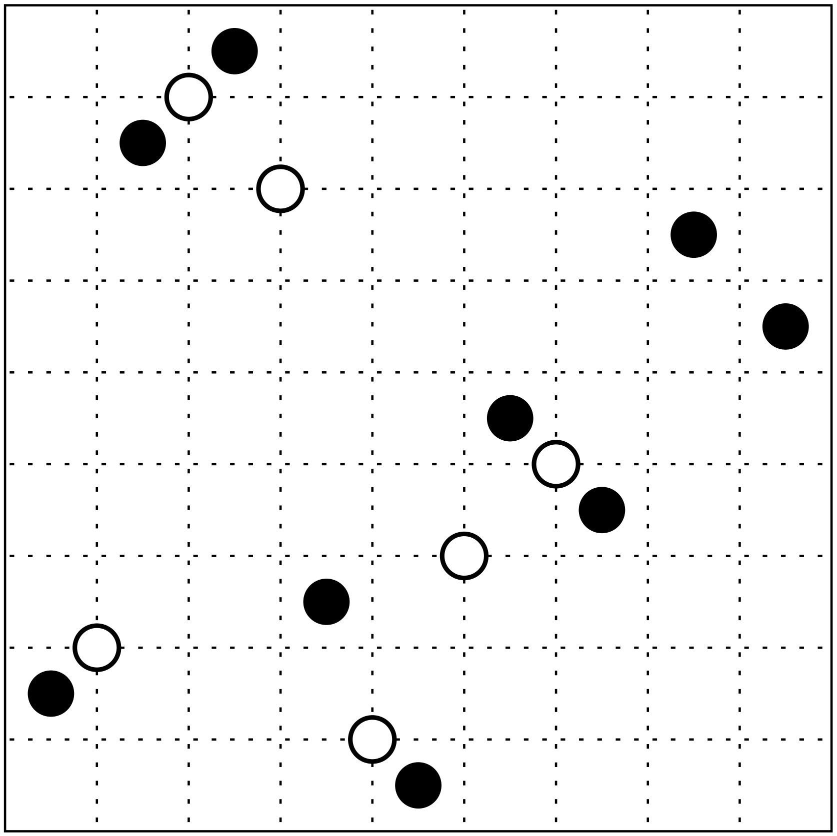

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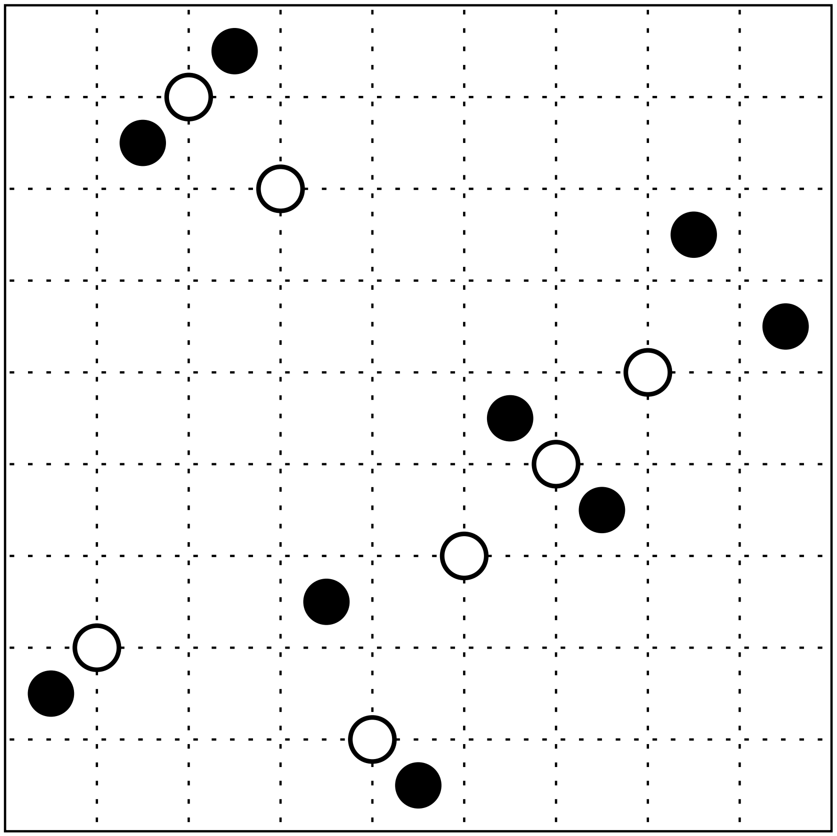

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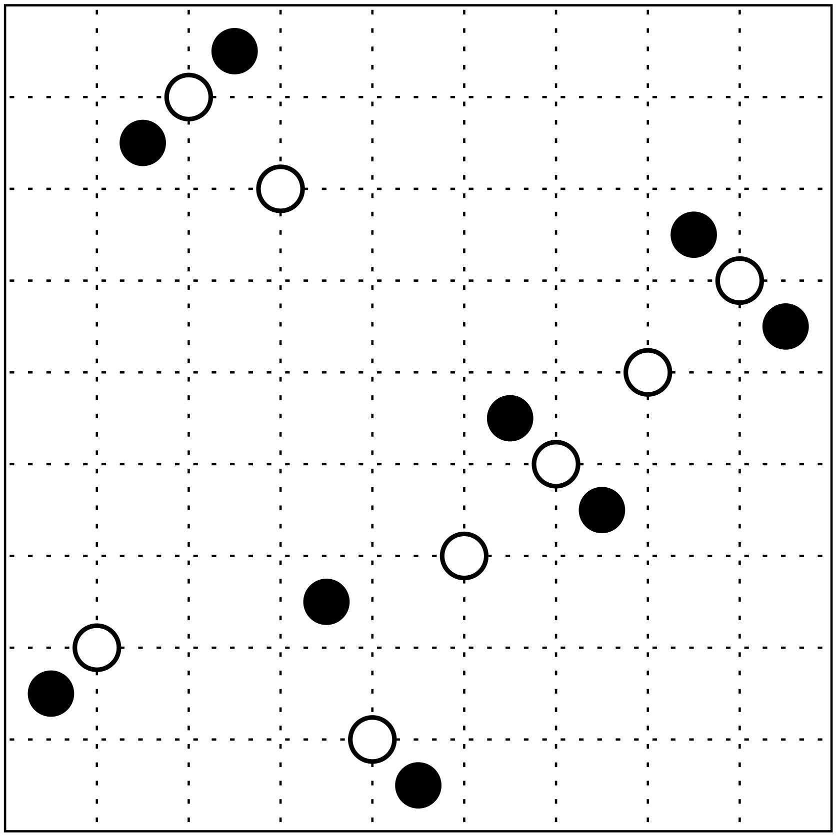

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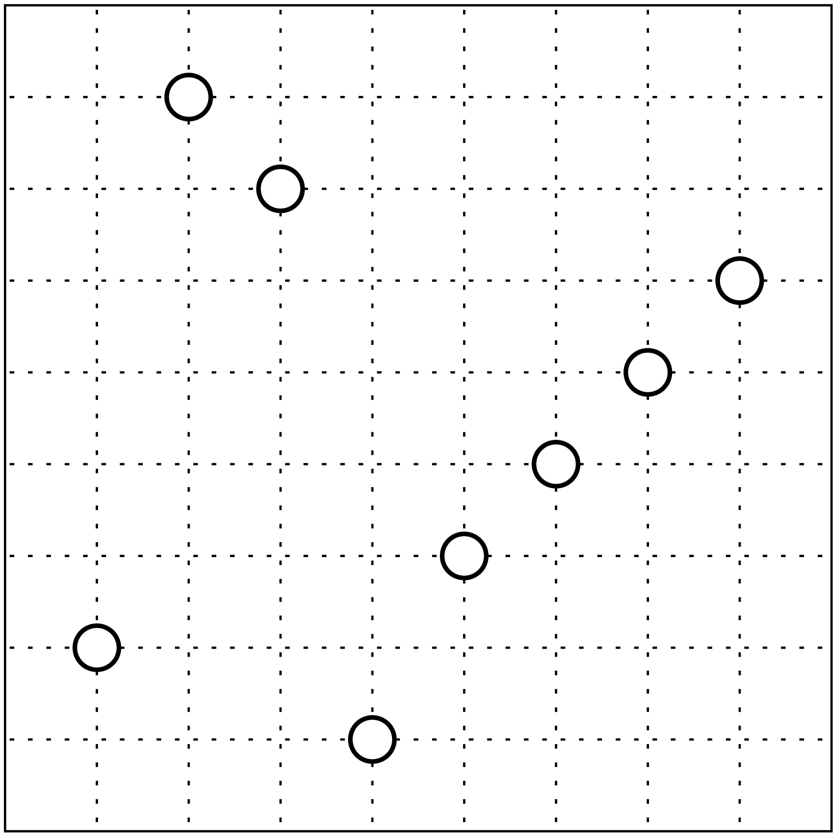
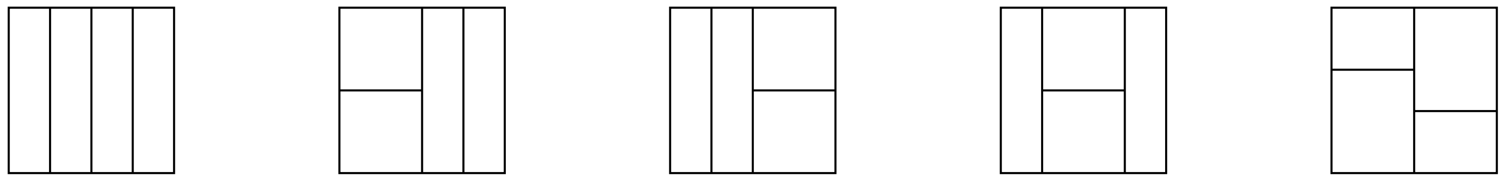
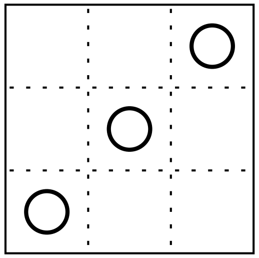


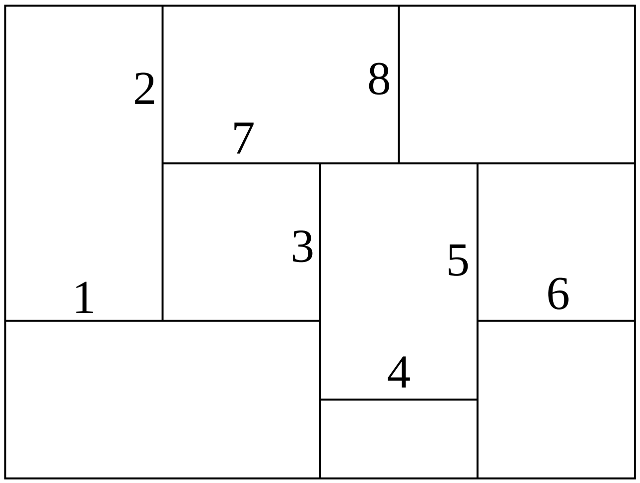
Floorplan partitions that give the same S-permutation

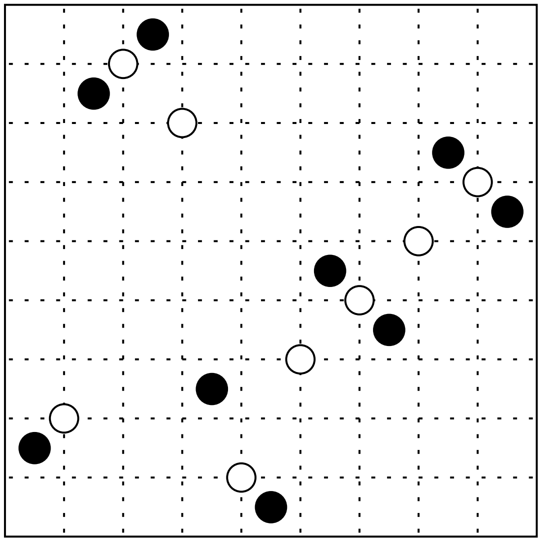
An F-block in a floorplan partition is a union of several rectangles which is a rectangle itself, and its inner partition is linear (the S-permutation is I or $-I$). For a fixed n, there are $2F_n$ F-blocks.



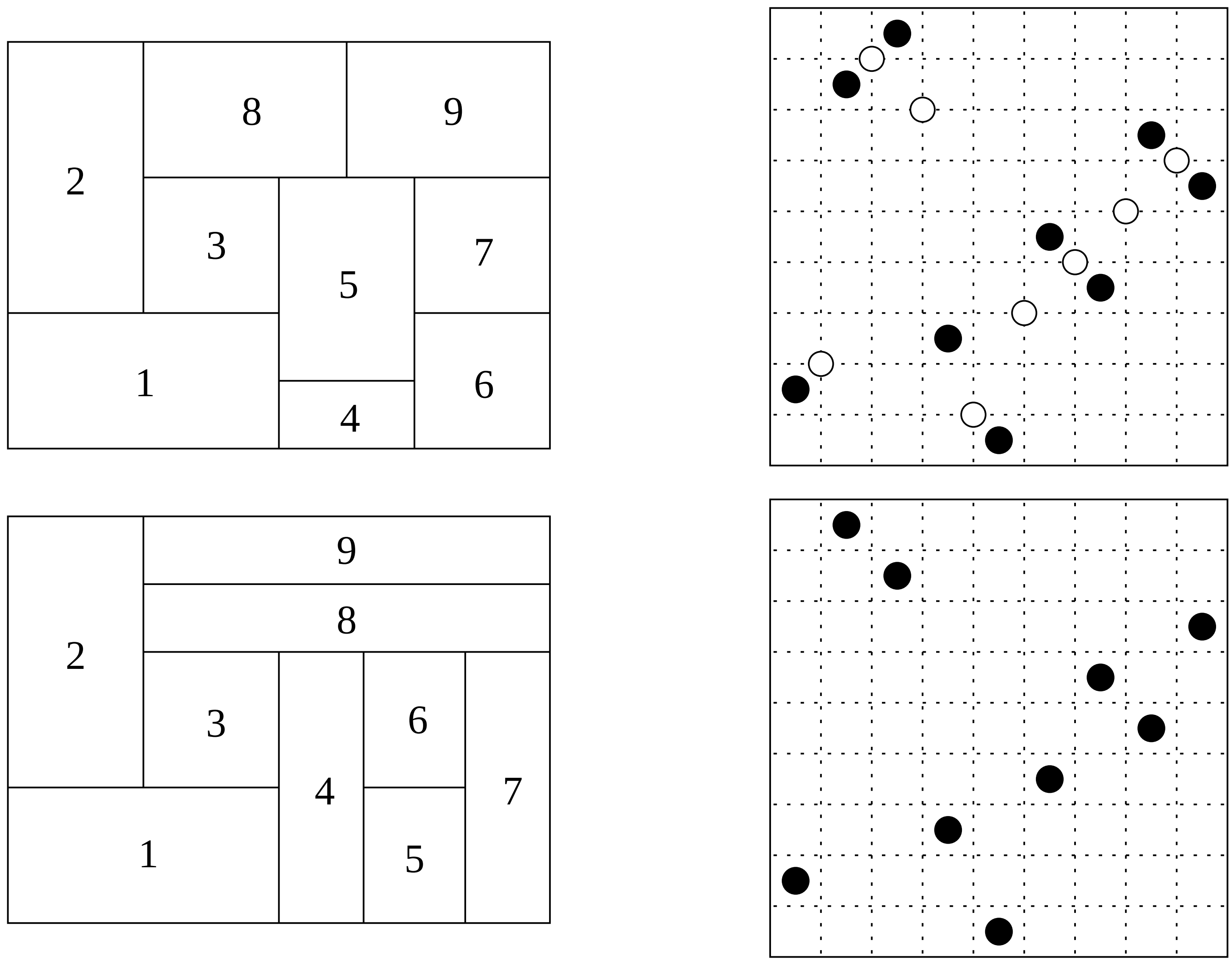


Floorplan partitions that give the same S-permutation

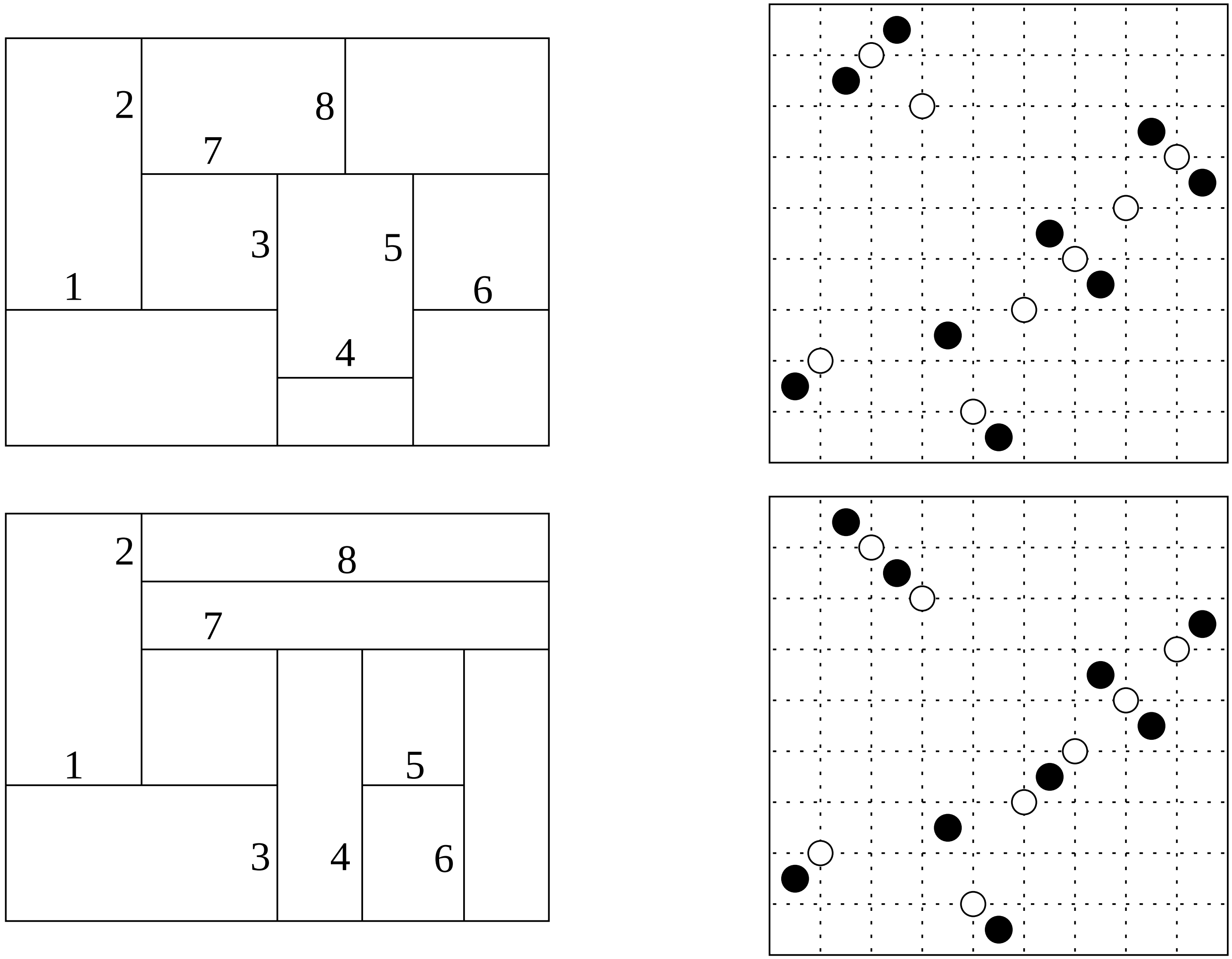




Floorplan partitions that give the same S-permutation

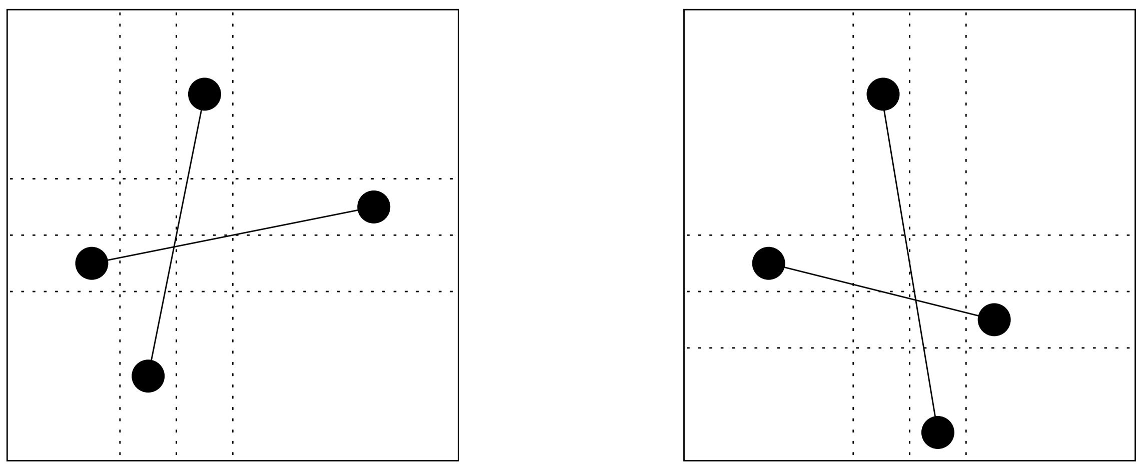


Floorplan partitions that give the same S-permutation



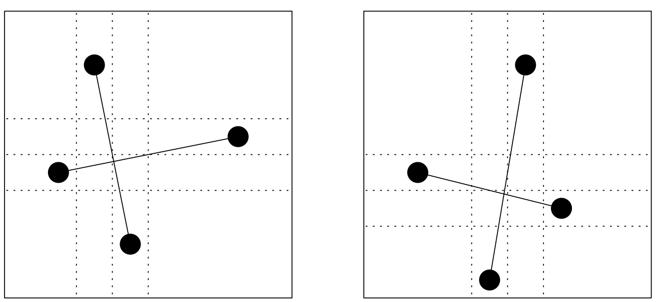
The family of all S-permutations

Theorem:
S-permutations are $(2-14-3, 3-41-2)$-avoiding
(or: $(21\bar{3}54, 45\bar{3}12)$-avoiding) permutations.
This correspondence is a bijection.

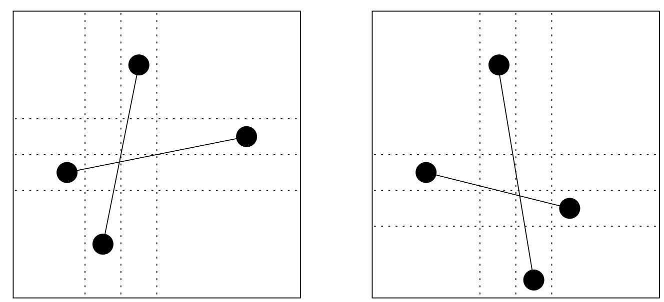


Floorplan partitions with the **rectangle** equivalence are in a bijective correspondence with $(2-41-3, 3-14-2)$-avoiding (or: $(25\bar{3}14, 41\bar{3}52)$-avoiding; or: Baxter) permutations.

Floorplan partitions with the **segment** equivalence are in a bijective correspondence with $(2-14-3, 3-41-2)$-avoiding (or: $(21\bar{3}54, 45\bar{3}12)$-avoiding) permutations.

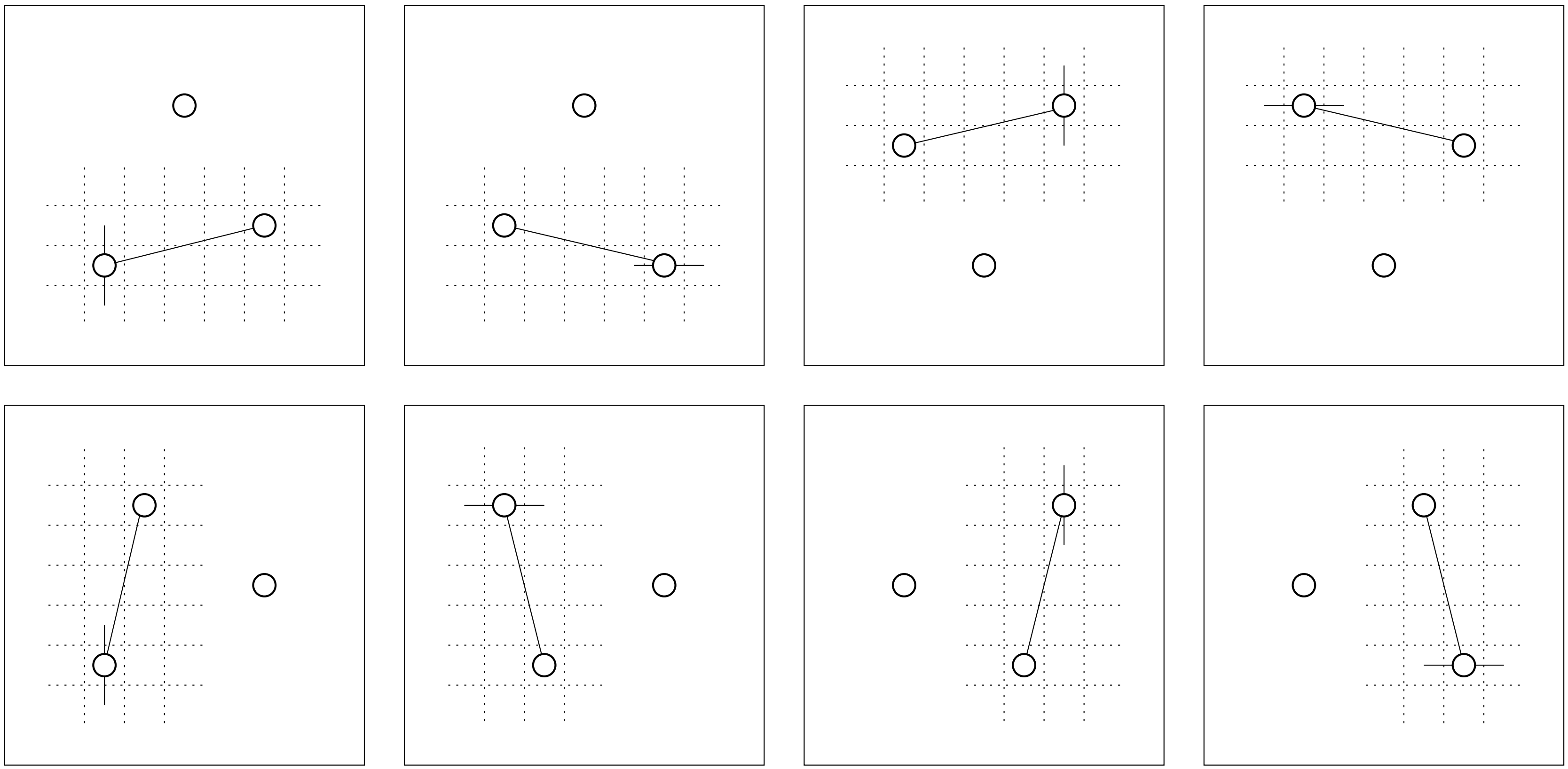


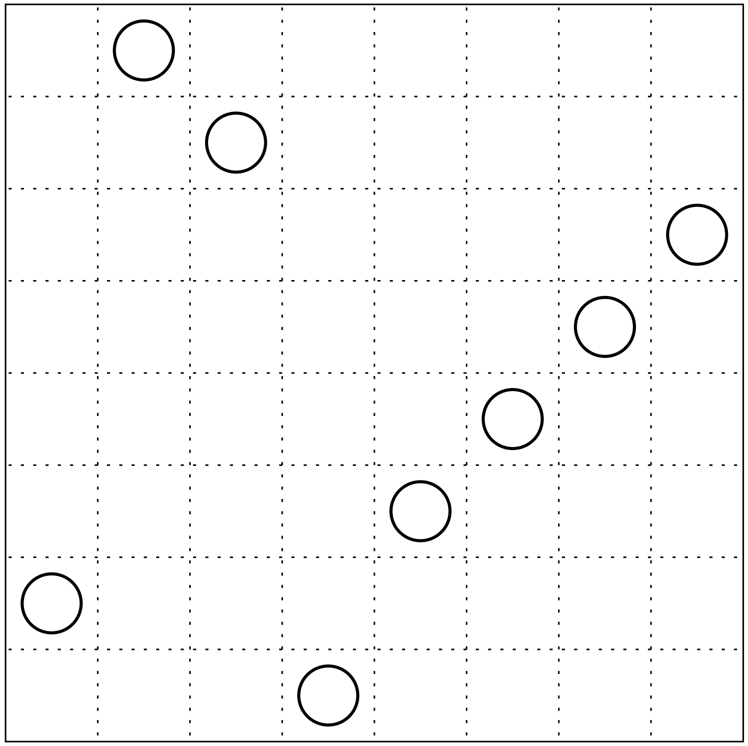
S(2−41−3, 3−14−2), Baxter

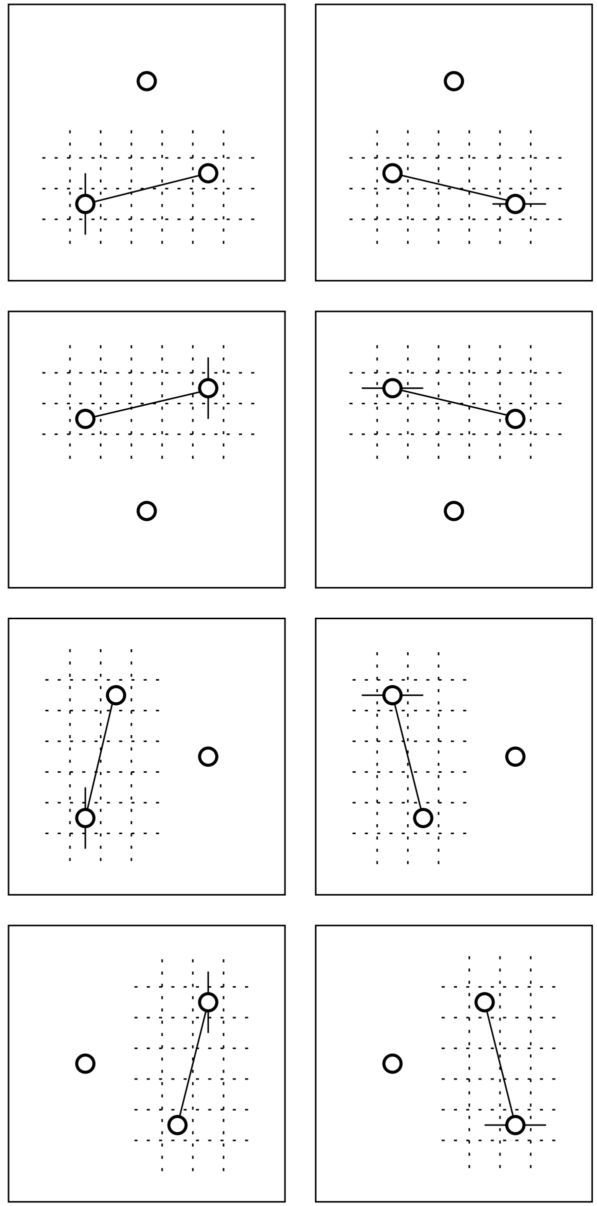


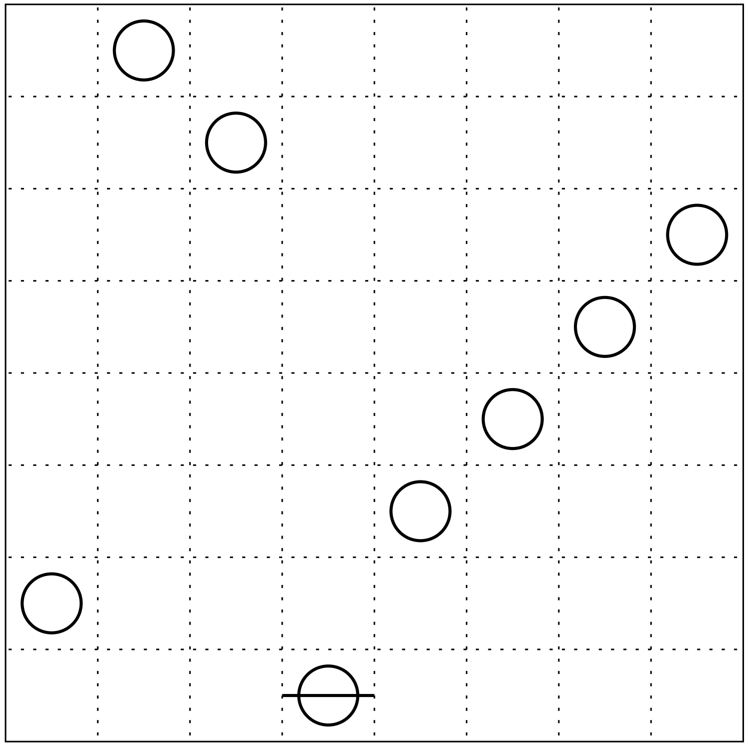
S(2−14−3, 3−41−2)

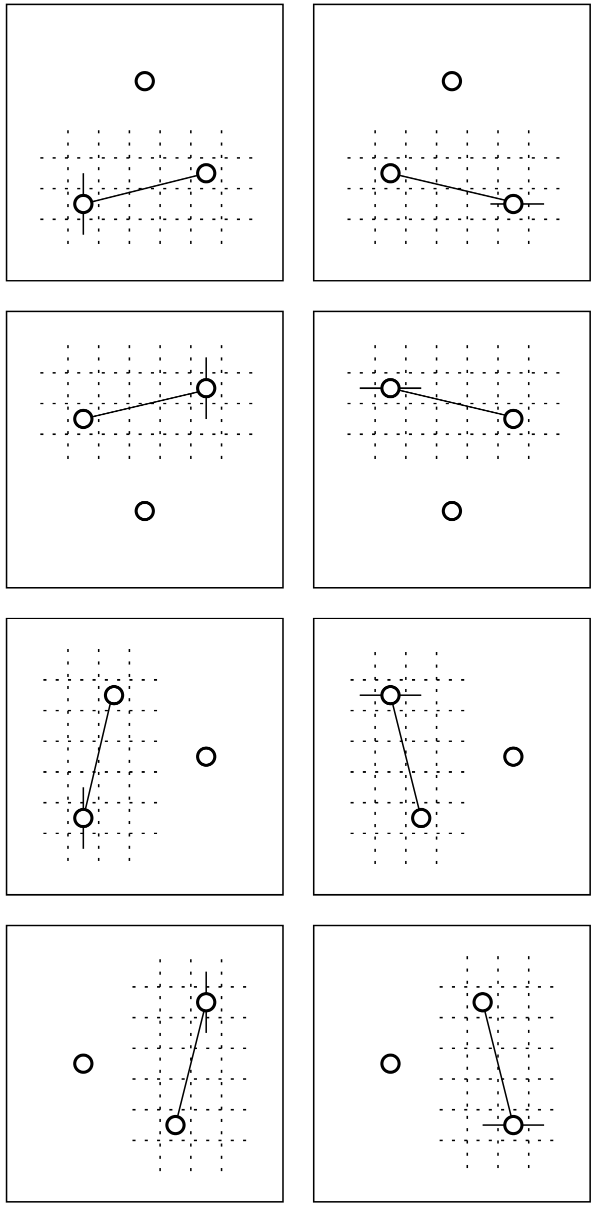
Reconstruction of a floorplan partition from its S-permutation

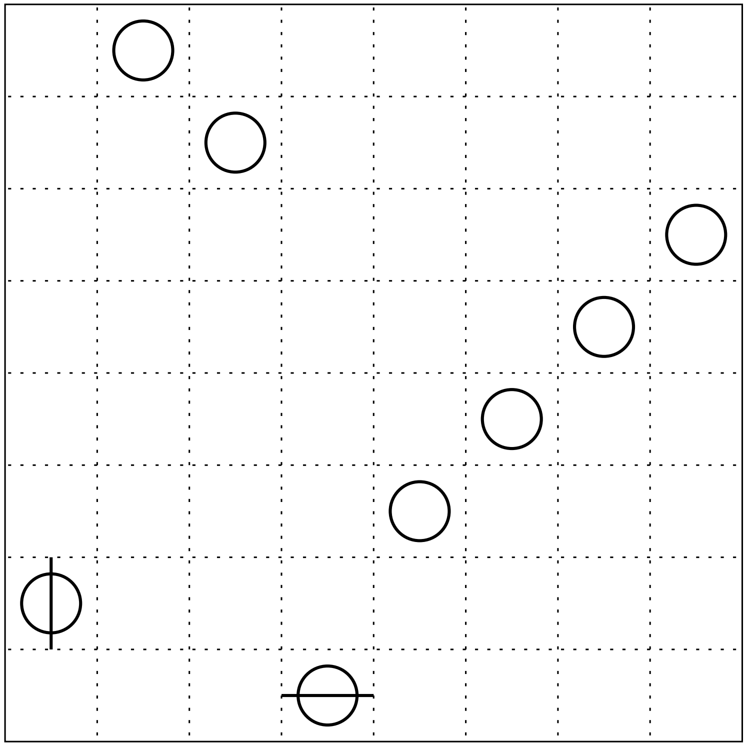


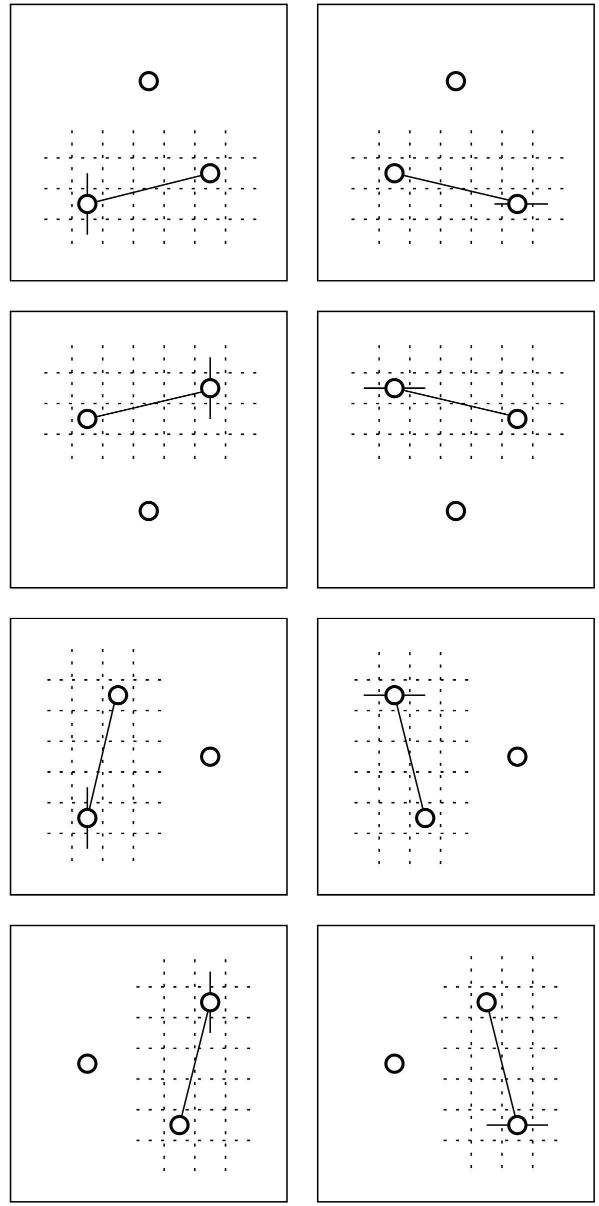


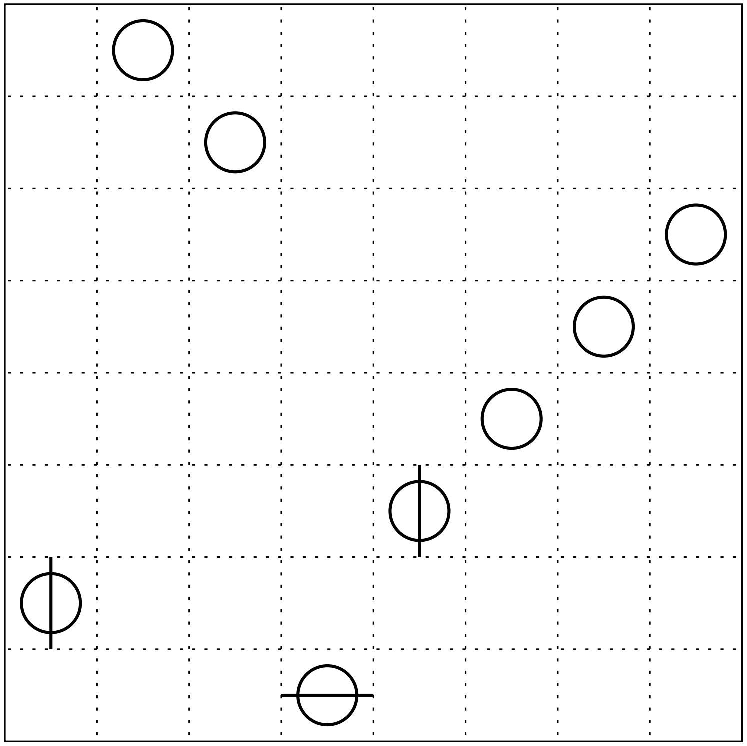


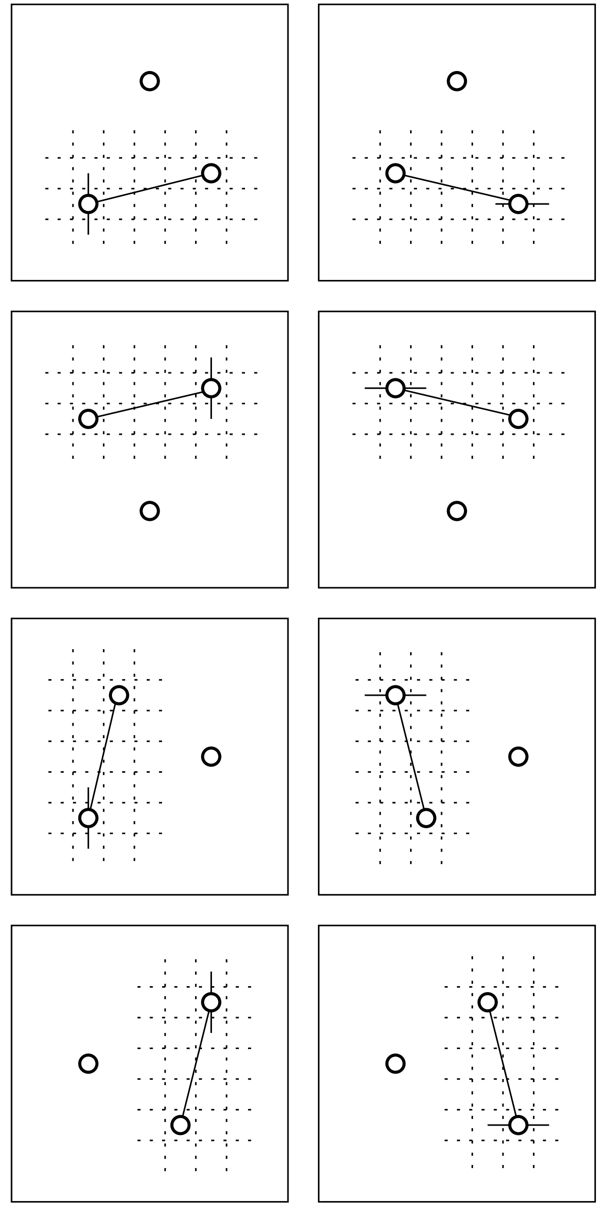


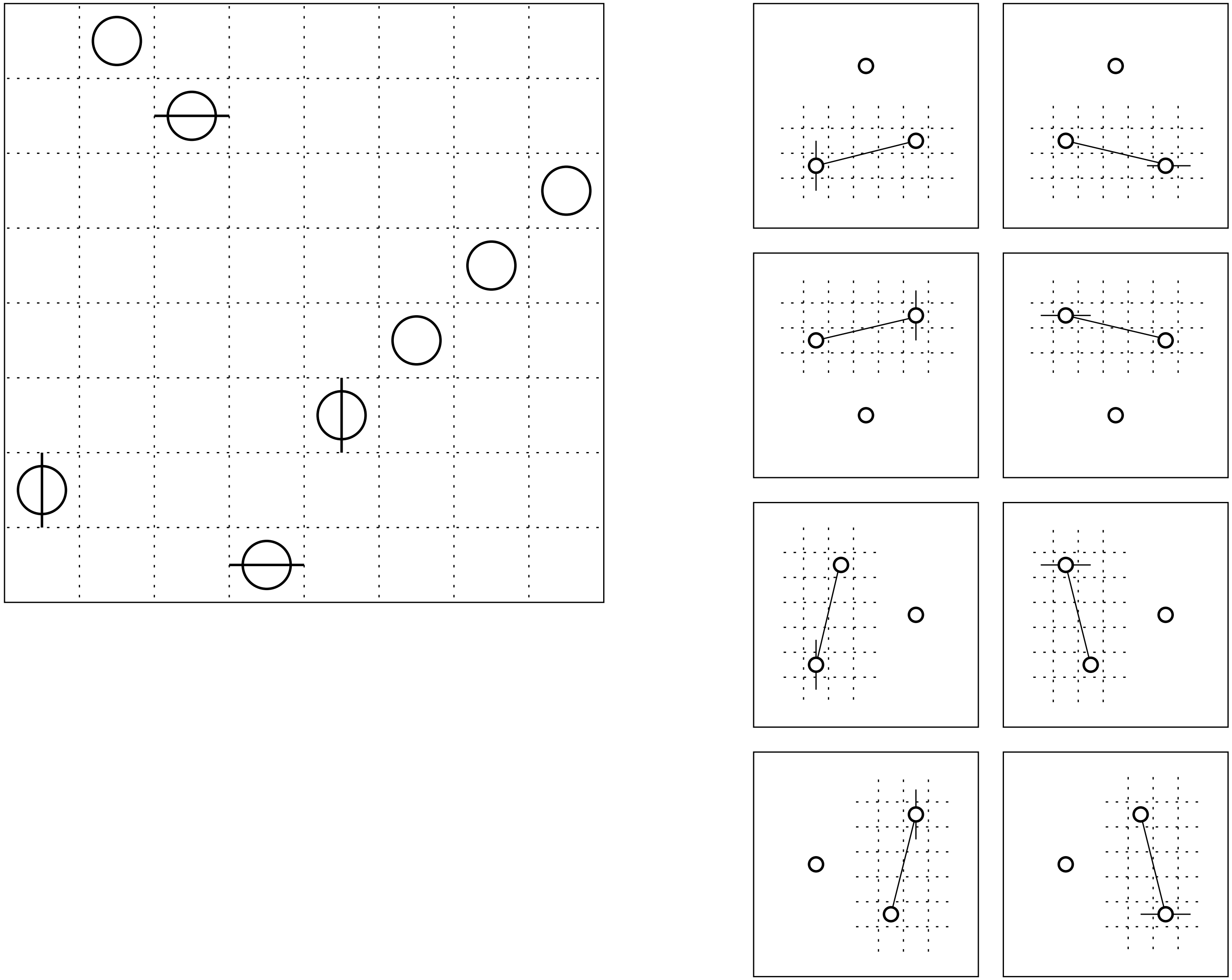


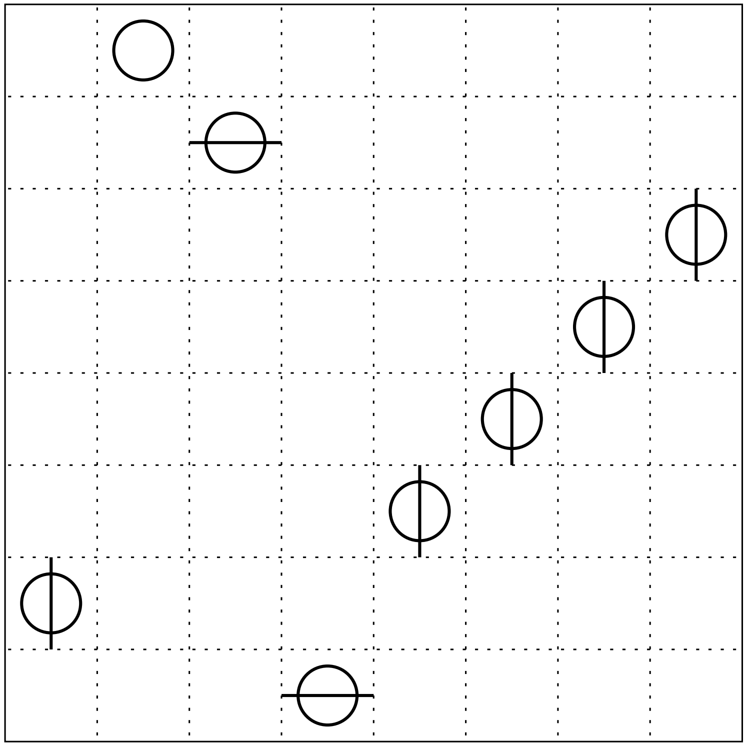
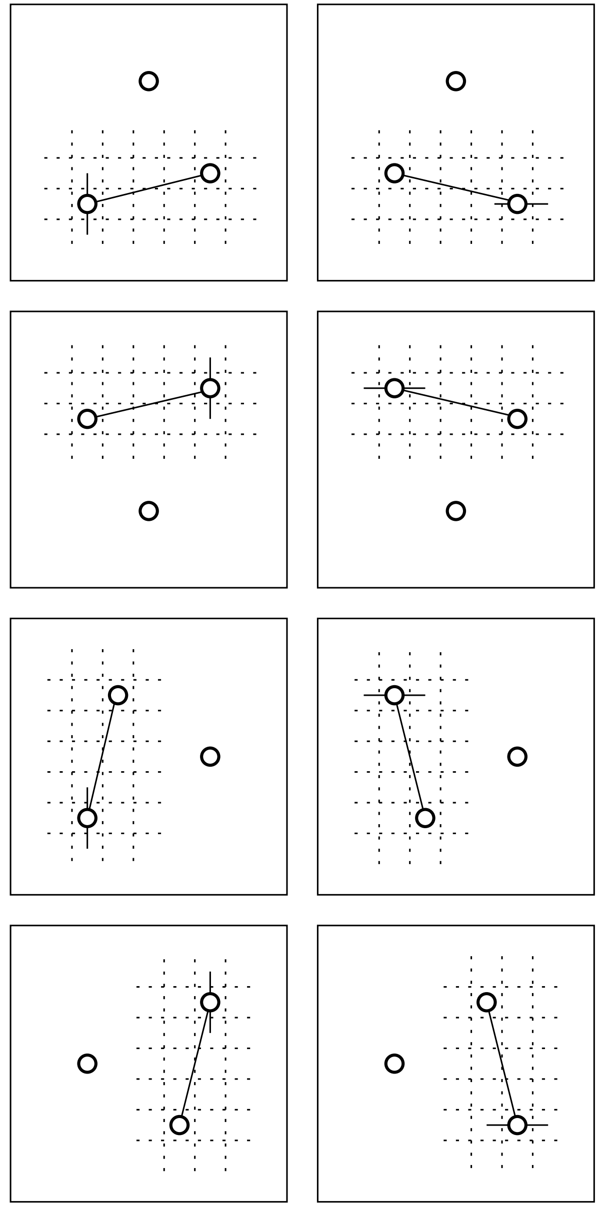


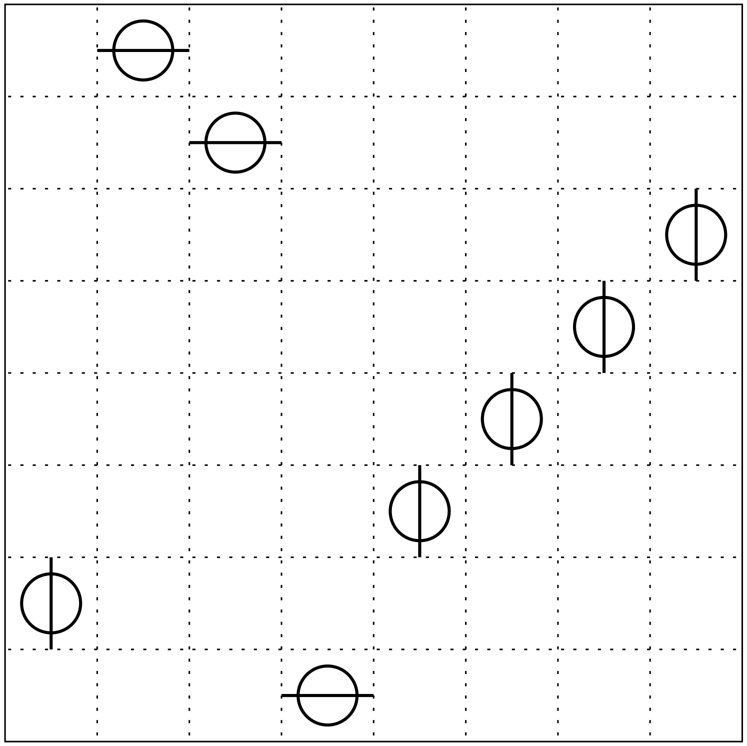
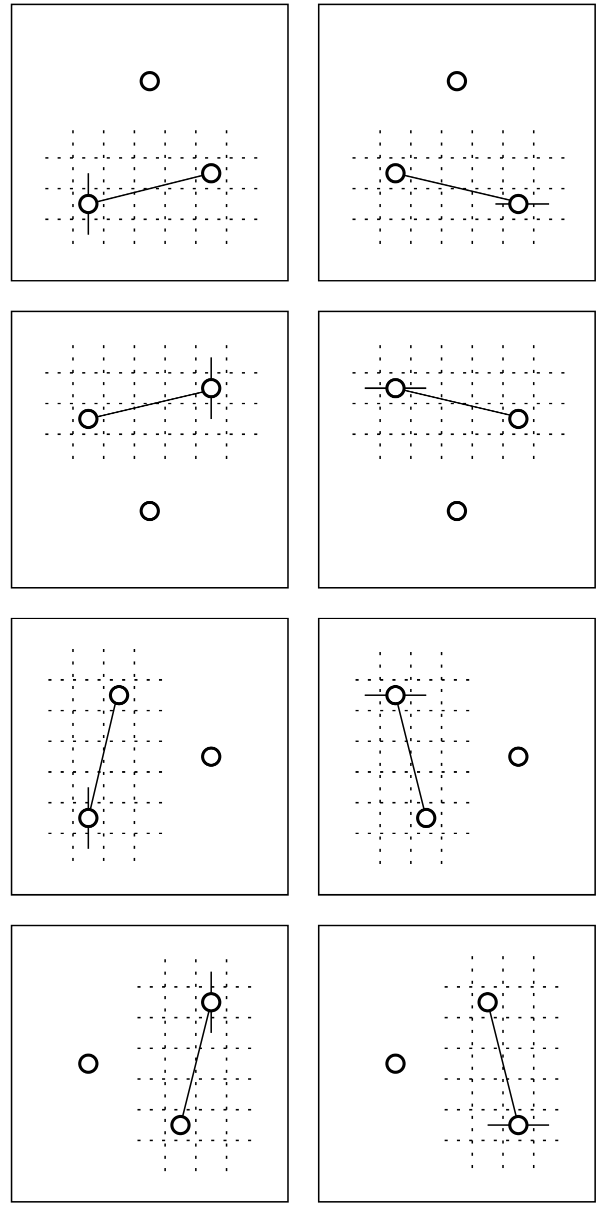


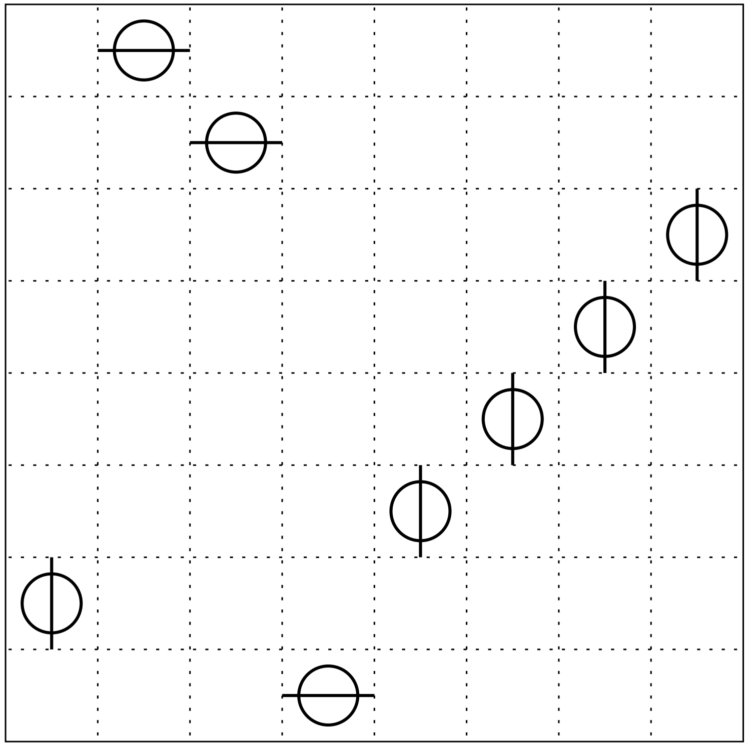
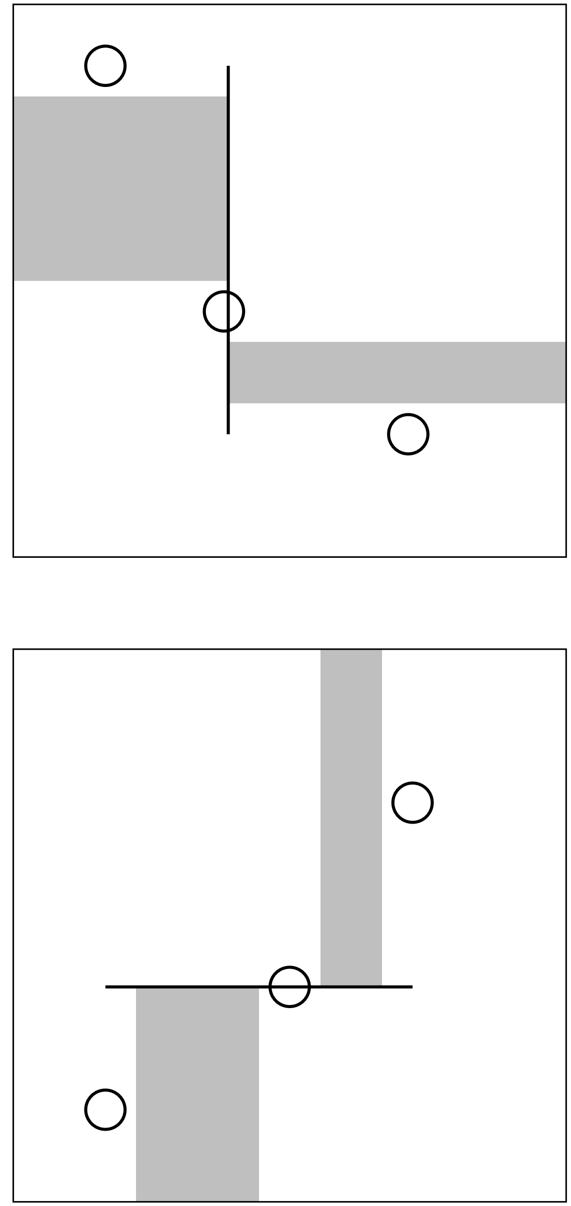


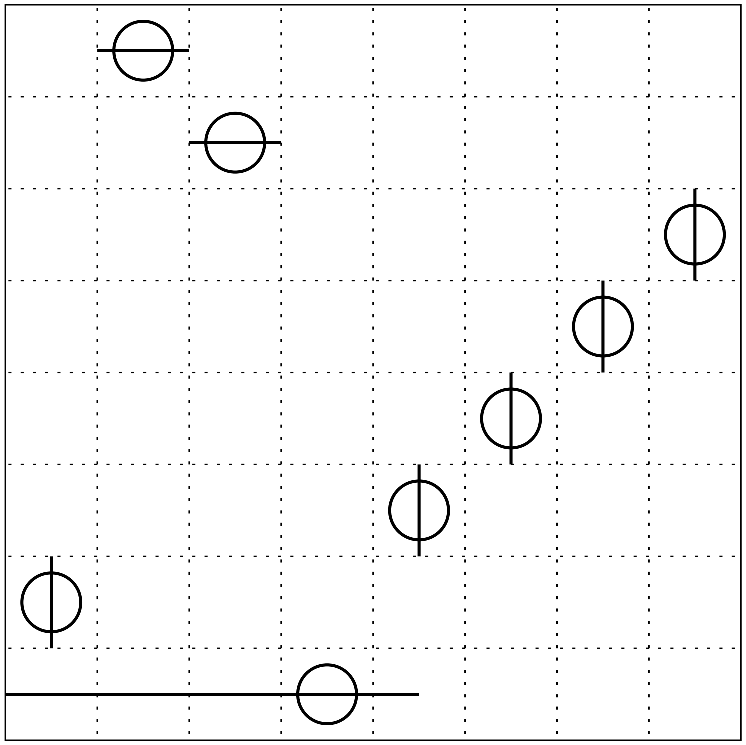


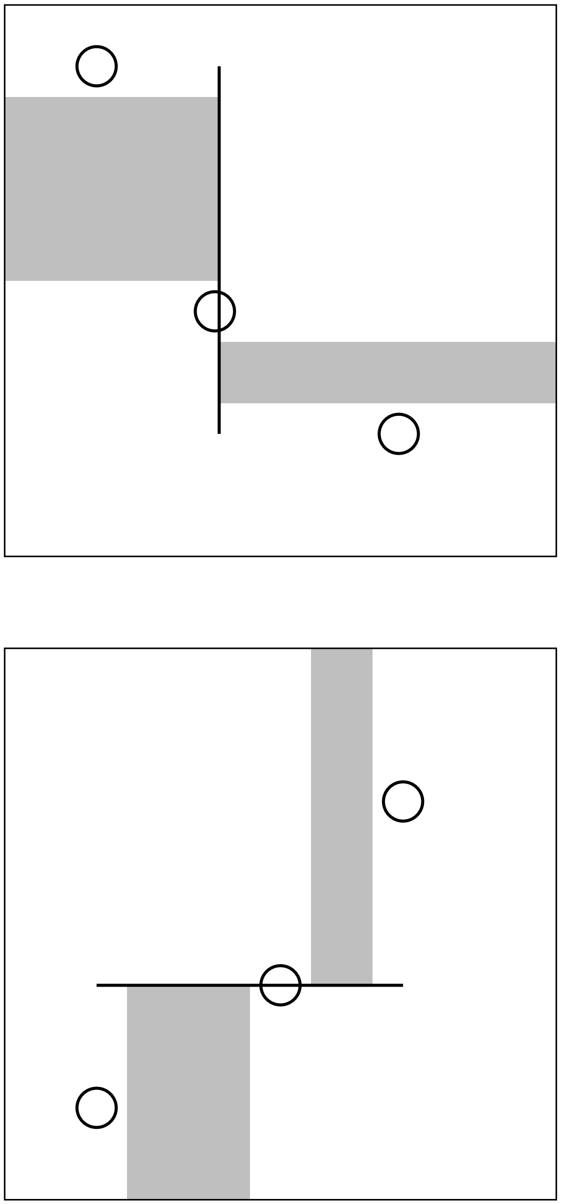


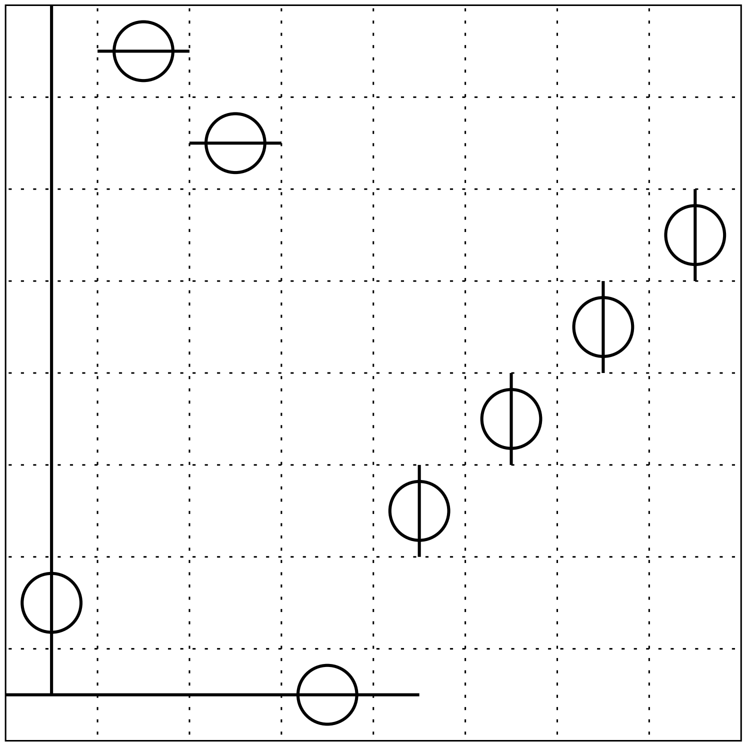



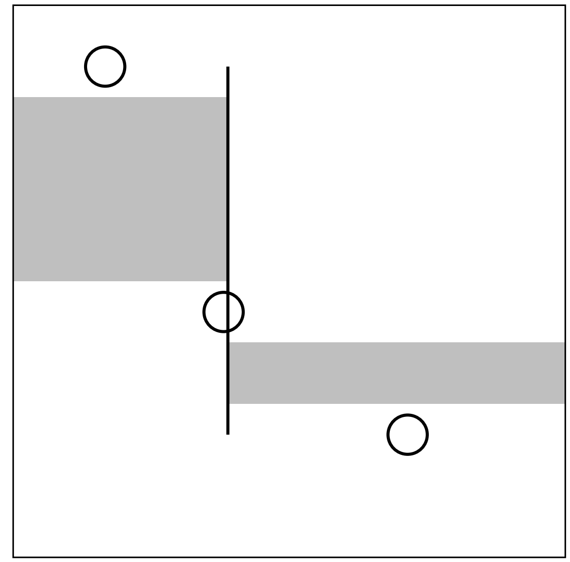



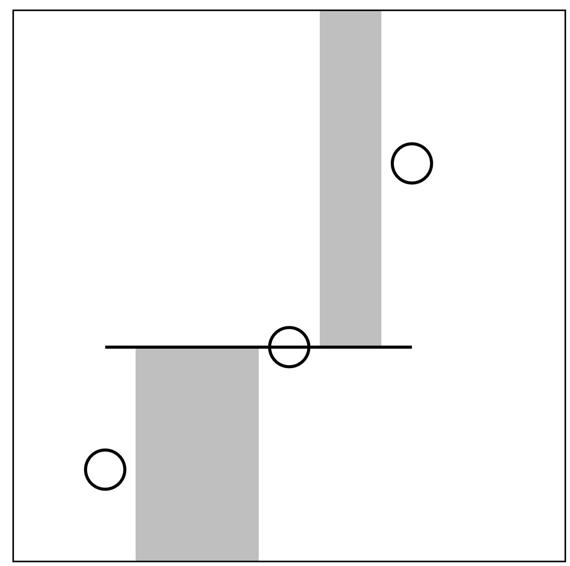



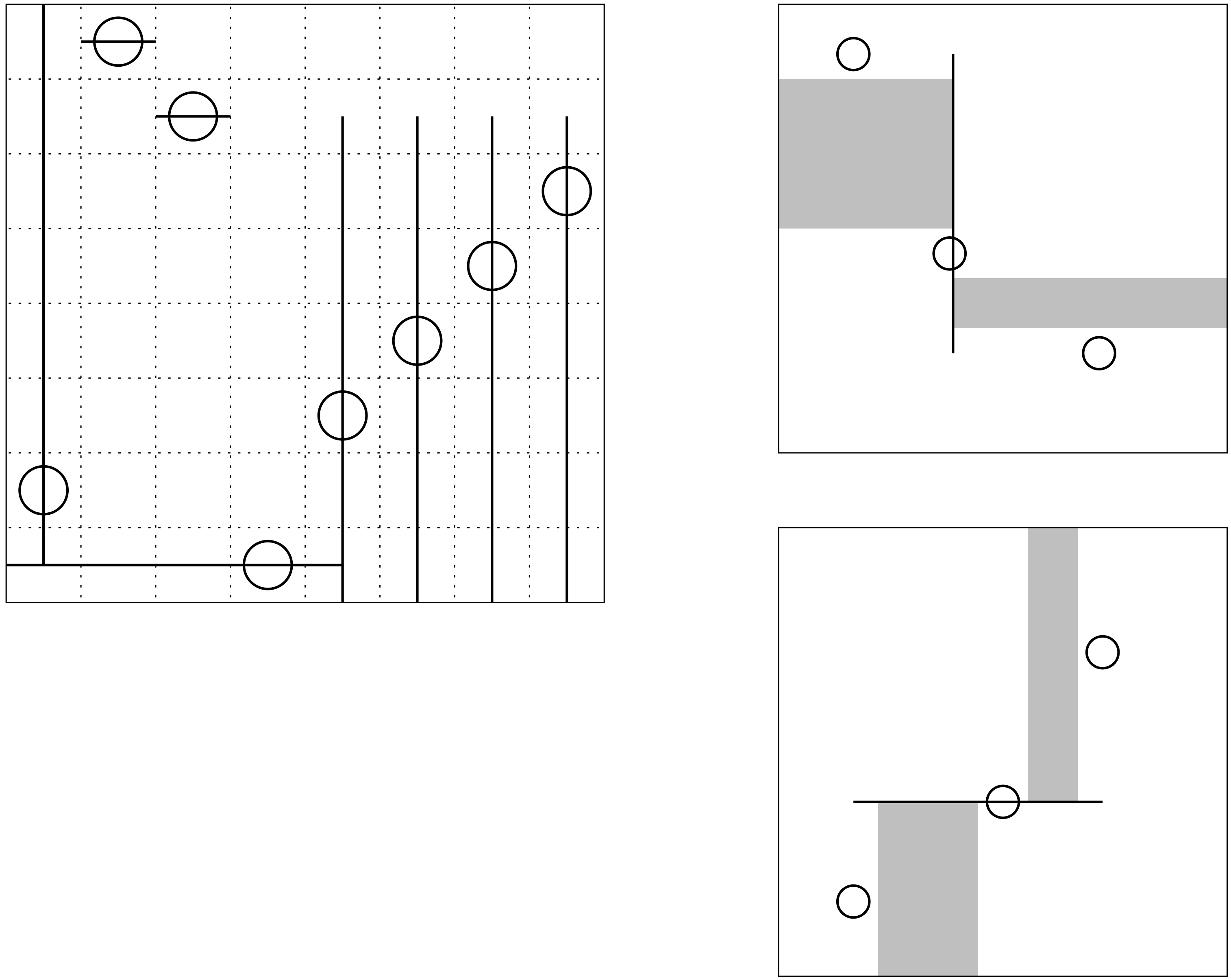


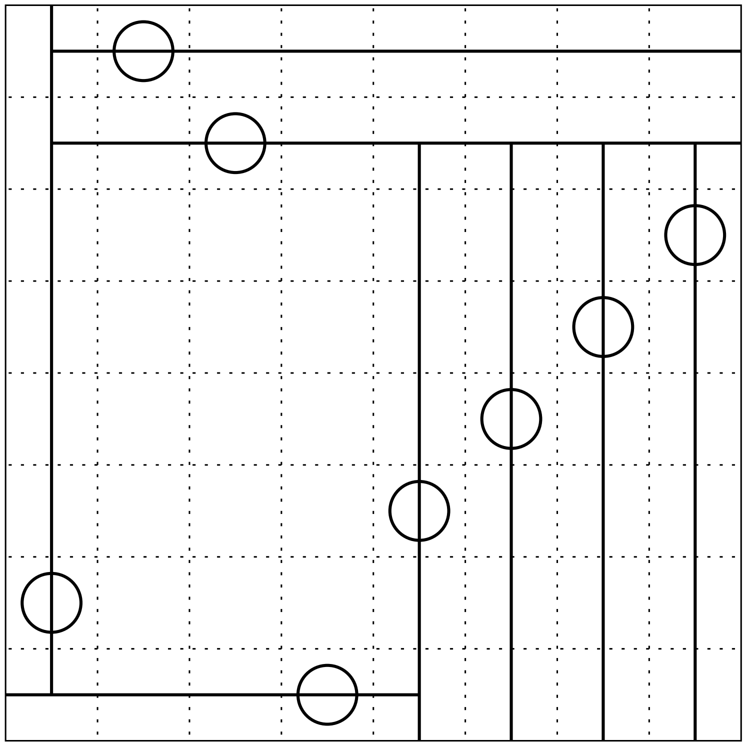


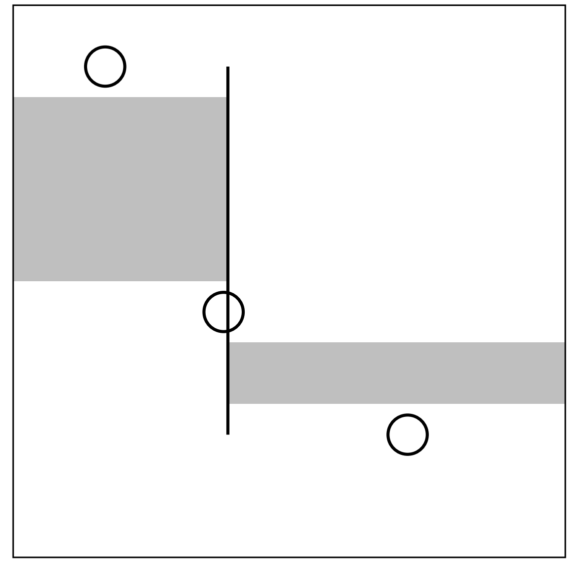


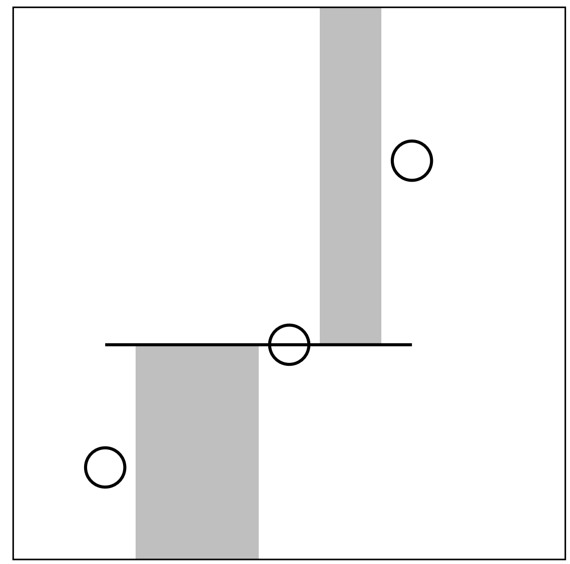


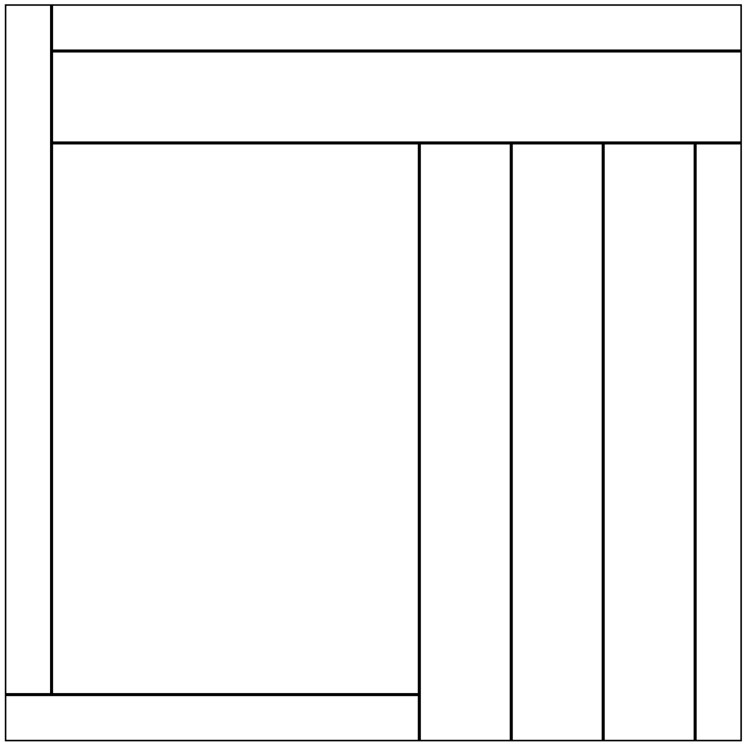


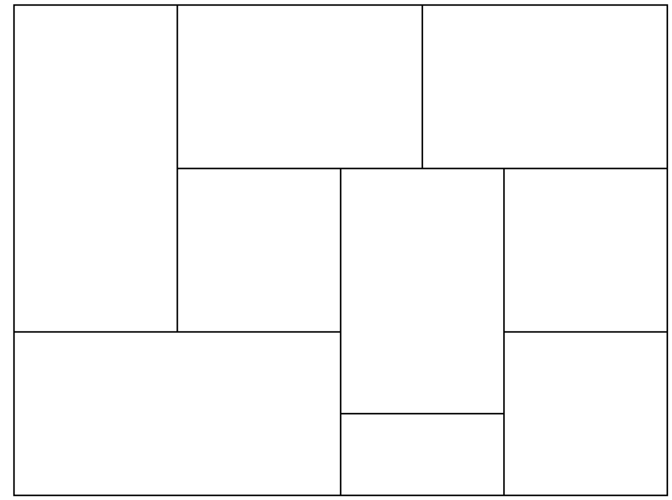




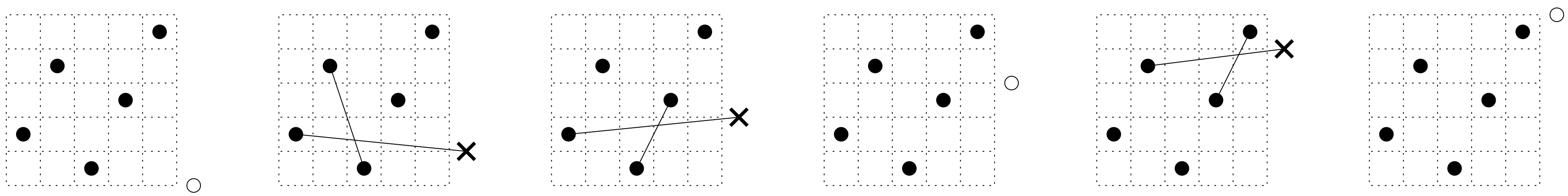


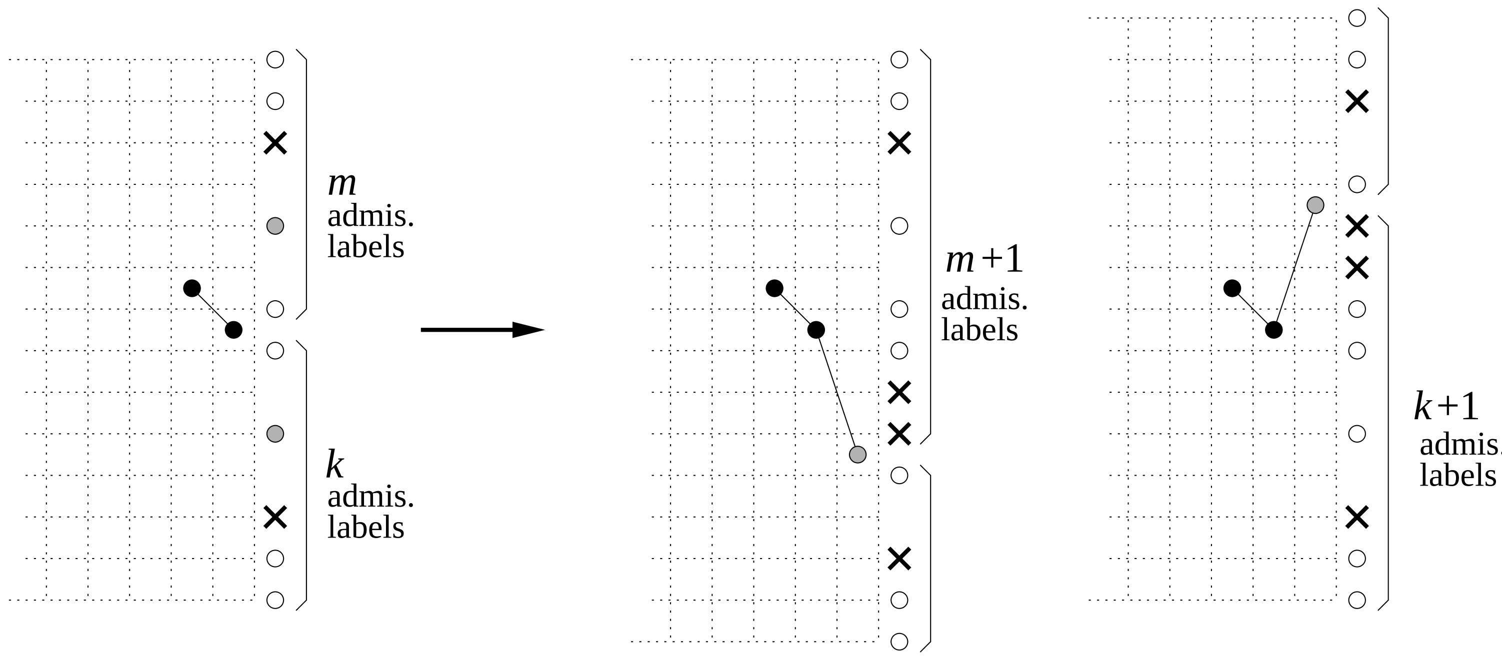






Generating tree for $(2-14-3, 3-41-2)$-avoiding permutations





(1) permutation π (● is an admissible point to be added)

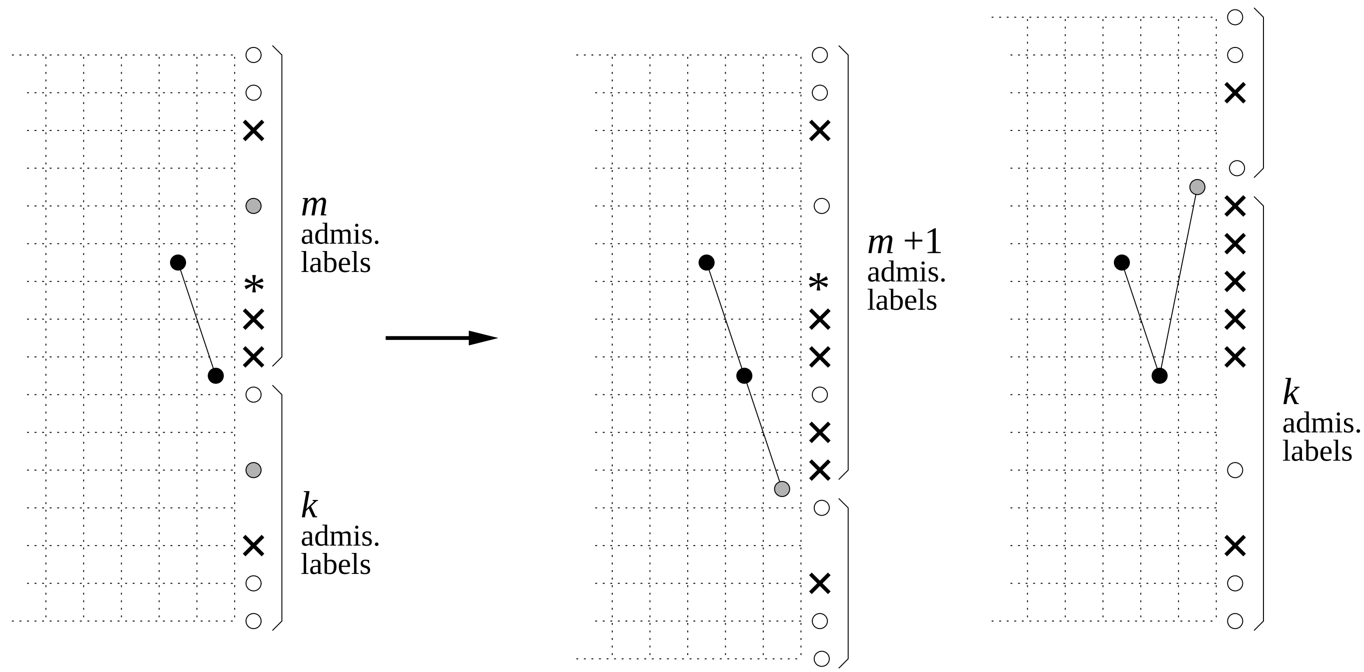
(2) permutation τ (a point below π(n) was added)

(3) permutation τ (a point above π(n) was added)

$$(k, m, s, d) \rightsquigarrow$$

$$(1, m+1, \ell, d),\quad (2, m+1, \ell, d),\quad \ldots,\quad (k-2, m+1, \ell, d),\quad (k-1, m+1, \ell, d),\quad (k, m+1, s, d);$$

$$(k+1, m, s, u),\quad (k+1, m-1, \ell, u),\quad (k+1, m-2, \ell, u),\quad \ldots,\quad (k+1, 2, \ell, u),\quad (k+1, 1, \ell, u).$$



① permutation π ② permutation τ (a point below π(n) was added) ③ permutation τ (a point above π(n) was added)

$$(k, m, \ell, d) \rightsquigarrow$$

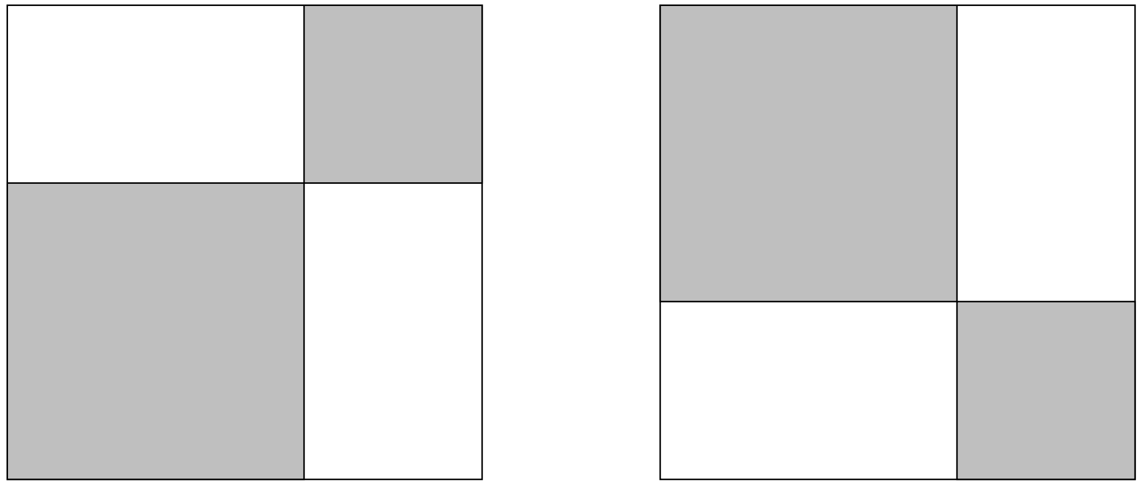
$$(1, m+1, \ell, d),\quad (2, m+1, \ell, d),\quad \ldots,\quad (k-2, m+1, \ell, d),\quad (k-1, m+1, \ell, d),\quad (k, m+1, s, d),$$

$$(k, m, \ell, u),\quad (k, m-1, \ell, u),\quad (k, m-2, \ell, u),\quad \ldots,\quad (k, 2, \ell, u),\quad (k, 1, \ell, u).$$

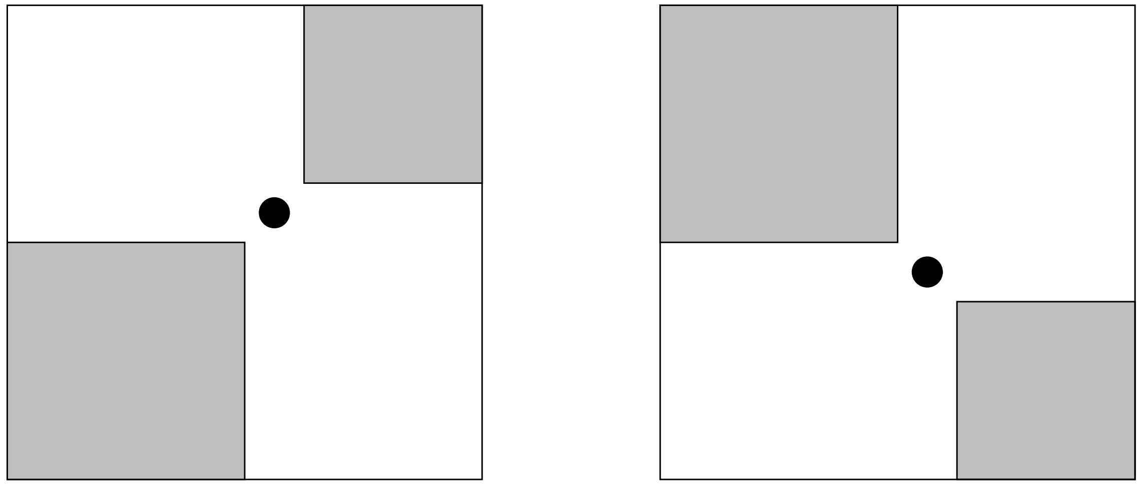
Enumerating sequence
for $(2-14-3, 3-41-2)$-avoiding permutations:

$1, 2, 6, 22, 88, 374, 1668, 7744, 37182, 183666, \ldots$

Separable permutations



Separable-by-point permutations



R-ordering:

All floorplan partitions $\leftrightarrow S(2-41-3, 3-14-2)$
(Baxter).

Guillotine partitions $\leftrightarrow S(2-4-1-3, 3-1-4-2)$
(Separable, Schröder).

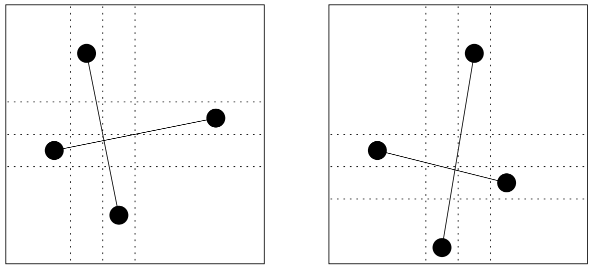
S-ordering:

All floorplan partitions $\leftrightarrow S(2-14-3, 3-41-2)$.

Guillotine partitions $\leftrightarrow$
$S(2-14-3, 3-41-2, 2-4-1-3, 3-1-4-2)$
(Separable by point).

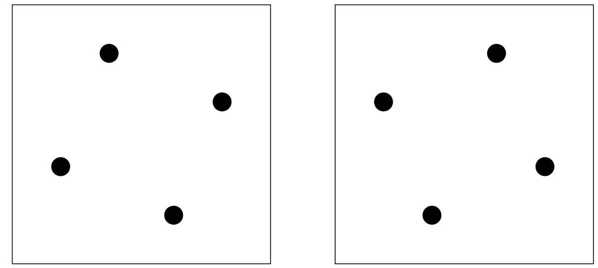
R−permutations:

All



S(2−41−3, 3−14−2), Baxter

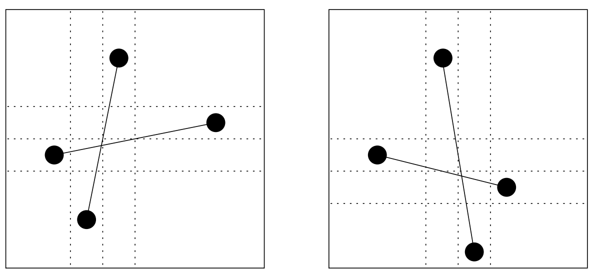
Guillotine



S(2−4−1−3, 3−1−4−2), separable

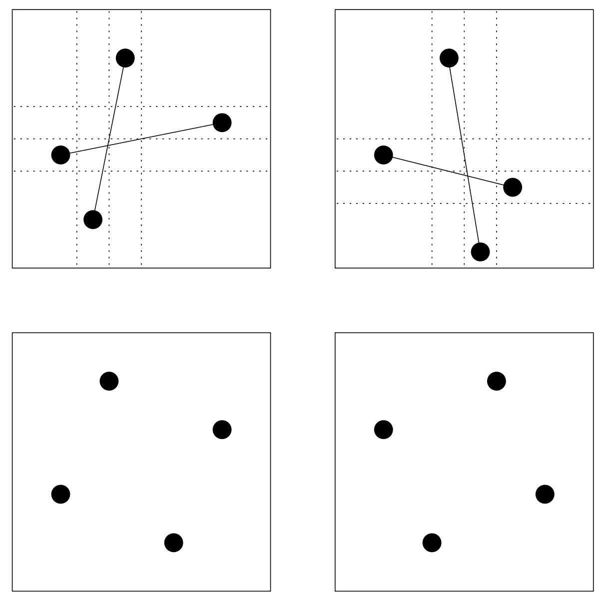
S−permutations:

All



S(2−14−3, 3−41−2)

Guillotine



S(2−14−3, 3−41−2, 2−4−1−3, 3−1−4−2), separable by point