Fuzzy predictive control for nitrogen removal in biological wastewater treatment

S. Marsili-Libelli and L. Giunti
Dept. of Systems and Computers, University of Florence, via S. Marta, 3, 50139 Florence, Italy. (E-mail: marsili@ingfi1.ing.unifi.it)

Abstract Whenever the carbon/nitrogen ratio of a domestic wastewater is too low, full denitrification is difficult to obtain and an additional source of organic carbon has to be provided. Since loading conditions may vary appreciably over the diurnal cycle, depending on the weather and sewage conditions, dosing should be controlled by an adaptive regulator to keep into account the time-varying process dynamics. A fuzzy predictive controller is proposed in this paper and its performance is tested through numerical simulations. The new aspects brought forward are the use of an improved model for denitrification, the use of benchmark (i.e. thoroughly tested and standardised) input files and the conclusion about regulator performance in overall plant performance, in terms of carbon saving and discharge compliance.

Keywords Adaptive control; denitrification; fuzzy control; model predictive control; wastewater treatment plant

Introduction

The problem of improving the nitrogen removal process from wastewater is considered. This problem is difficult to solve if the carbon/nitrogen ratio is low, as often happens with domestic sewage where a considerable part of the carbon loading is removed by septic tanks before the waste is routed to the WWT plant. If the wastewater does not contain enough organic carbon, denitrification may be impaired and an external carbon source is required. The aim of this paper is the design of a real-time controller to perform an efficient and economical carbon dosing to maximise nitrogen removal through denitrification and minimise the additional operational cost. The study is based on the simulation of an appropriate model and control strategy. Given the uncertain nature of the process and its variability, an adaptive controller is considered. Since incoming nitrogen and carbon flows constitute the main disturbances driving the system, a predictive controller has been considered, in which process nonlinearities and control constraints can be easily incorporated. From a practical point of view, the study is primarily aimed at assessing the controller performance in its economic dimension, i.e. finding the minimum amount of carbon needed for denitrification. One of the results of the research is an indication of how the carbon set-point should be specified in order to comply with discharge standards imposed by the EC directive 271/1991 and still save money. The results are presented in terms of numerical simulation, since the control system is now in the process of being ported into a pilot plant for field trials. Though the feedback control of several WWTP operations has been considered before by several researchers, notably Lindberg (1997) and more recently Olsson and Newell (1999), a new controller is presented in this paper with a Model Predictive Controller (MPC) structure. Its new features include the use of a Sugeno predictive model and a numerical procedure to solve the optimisation problem inherent in the MPC with a Branch and Bound (B&B) approach. As in a previous paper (Marsili-Libelli and Manzini, 2000) an improved model for denitrification derived from the ASM2d model (Henze et al., 2000) was used, together with benchmark (i.e. thoroughly tested and standardized) input files (Alex et al., 1999).
As to the model, the basic ASM2d has some limitations which make it unsuitable to describe the denitrification process in details. The model used in these simulations, derived from the ASM2d, is described in Marsili-Libelli et al. (2001). In particular, the nitrification phase has to be enhanced with nitrite dynamics. Further, the dynamics of phosphorus accumulating organisms (X\text{PAO}) was improved, taking into account the denitrification capability of these organisms, in competition with other less specialised heterotrophs. These model improvements are described in detail in a previous paper, where deterministic adaptive controllers were used (Marsili-Libelli and Manzini, 2000). In particular, the modified model includes two separate populations of ammonia oxidisers and nitrite oxidisers, each with separate decay coefficients, yield factors and maximum growth rates. The model kinetics include: anoxic growth on both fermentable and fermented substrate, anoxic uptake of polyphosphate and anoxic growth of phosphorus accumulating organisms (PAO).

The modified ASM2d model for the denitrification process is too complex to be used as a basis for controller design. In addition to complexity, moreover, the model is time-varying given the fluctuation in process flow rate and organic loading, inducing variations in all the process parameters. Hence the need to use a simplified model for controller design and operation. In the present context, a fuzzy model is considered which is tuned during process operation, as described in the following.

**The problem of carbon dosing in denitrification**

The process scheme adopted for this study is shown in Figure 1. It reproduces the structure of a pilot plant used to simulate the operation of the City of Florence WWTP which is about to start operating. The pilot plant will be used to test control strategies prior to large-scale plant implementation. It includes an anoxic tank for denitrification, followed by an aerobic tank for nitrification and organic carbon oxidation. The last tank is a small anaerobic chamber for complete deoxygenation prior to recycling the mixed liquor to the anoxic tank. There are two recycle streams: \( Q_{\text{int}} \) recycles the nitrate-rich effluent into the denitrification tank, whereas \( Q_{\text{r}} \) is the sludge recycling flow from the settler. The carbon dosing control scheme is also shown in Figure 1.

The aim of the study is to design an efficient control strategy for carbon dosing in the anoxic tank where denitrification occurs. As shown in the previous paper (Marsili-Libelli and Manzini, 2000), this process is the sum of several concurrent kinetics in the anoxic tank and involves heterotrophic growth on fermented and fermentable substrates, anoxic uptake from nitrate and anoxic growth of PAO on nitrates. All these kinetics are of course altered by the time-varying dilution rate \( Q_{\text{in}}/V_{\text{ANOX}} \) during process operation.

![Figure 1](image-url)
Controller structure

Predictive Control (PC) has been extensively researched and in the linear case without input constraints its solution is the well known linear quadratic (LQ) problem, which can be solved via the Riccati equation. When the process dynamics is nonlinear and there are input constraints, the problem complexity increases considerably (Allgöwer and Zheng, 2000), analytical solutions are difficult to find and a numerical approach is preferred. The PC algorithm presented in this paper is based on fuzzy models and a Branch and Bound numerical search to determine the optimal control sequence. The basic idea of predictive control is to test a collection of possible input over the control horizon $H_c$, after which the input $u$ is held constant, and evaluate the output $y$ over the prediction horizon $H_p$ using the process model, as shown in Figure 2. The FPC used in this application and shown in Figure 2 has the typical Internal Model Control structure (Morari and Zafiriou, 1989) in the sense that the feedback signal is the error between process output and model prediction $e(k) = S_{NO_3}(k) - \hat{S}_{NO_3}(k)$. Two fuzzy models are used: a one-step-ahead model generating the output error which is fed back to the controller, and a $H_p$-step-ahead model for the output predictions from which the optimal control sequence $u^*$ is determined. For clarity, in the following the nitrate concentration $S_{NO_3}$ will be simply indicated by $y(k)$ and the dosing carbon flow $Q_{in}$ by $u(k)$. The superscript "$\hat{}$" denotes an estimated quantity.

Fuzzy models

Both models are based on a Sugeno structure in the sense that the output is a fuzzy combination of a set of $i (i = 1,\ldots, M)$ linear consequent models whose degree of activation $\mu_{i,k}$ is obtained by clustering the input/output data and not through fuzzy antecedents as in the original structure (Tagaki and Sugeno, 1985). This structure is now extensively used as a fuzzy predictive model for its simplicity (see e.g. Babuska, 1998). The input/output clustering is performed through the Fuzzy C-Means (FCM) algorithm (Bezdek, 1981) producing the degrees of activation $\mu_{i,k}$. These form the antecedent pair of the mode and are obtained from an input/output fuzzy similarity measure with respect to a set of prototypes previously determined during the training phase and which can be adapted during operation (Marsili-Libelli, 1998).

One-step-ahead predictor. This model is formed by a collection of linear models producing a one-step-ahead prediction based on past process outputs. The vector $z(k)$ is a collection of past inputs and outputs defining the operational regime of the process, which is partitioned into $M$ clusters $Z_i$ whose membership $\mu_{i,k}$ determines the degree of activation of the
corresponding consequent model. The memberships \( \mu_{i,k} \) are computed with the Fuzzy C-Means (FCM) clustering algorithm (Bezdek, 1981, Marsili-Libelli and Müller, 1996). In this way a major memory saving is obtained compared to the classical fuzzy rule-based antecedents. Model (1) can be concisely written as the sum of the linear consequents weighted by the corresponding activation membership, since FCM introduces the constraint \( \sum \mu_{i,k} = 1 \)

\[
\begin{align*}
(1) & \quad \hat{y}(k+1) = \sum_{i} \hat{y}_{i}(k+1) \mu_{i,k} \\
& \quad \sum_{i} \mu_{i,k} = 1
\end{align*}
\]

where \( \hat{y}_{i}(k+1) = a_{i0}y(k) + \ldots + a_{i,n_{y}-1}y(k-n_{y}+1) + b_{0,i}u(k) + \ldots + b_{n_{u}-1,i}u(k-n_{u}+1) + c_{i} \) is the parameter vector of the \( i \)th model which is estimated with ordinary recursive least-squares at each time-step \( k \) and the model regressor \( \varphi(k) \) is composed of past inputs and outputs.

\( \varphi(k) = \left[ y(k)y(k-1)\ldots y(k-n_{y}+1)u(k)u(k-1)\ldots u(k-n_{u}+1) \right]^T \)

**H_p-step ahead predictor.** While the structure is basically the same as the previous model (1), the main difference is in the regressor composition. In fact, since this model should be able to predict the system behaviour \( H_p \) steps ahead of the current time \( k \), it makes use of all the intermediate predictions between time \( k \) and \( k+H_p \). Thus a modified regressor \( \hat{\varphi}(k+i) = \left[ \hat{y}(k+i) \hat{y}(k+i-1)\ldots \hat{y}(k) \ldots y(k-n_{y}+1) u(k+i) u(k+i-1)\ldots u(k) \ldots u(k-n_{u}+1) \right]^T \) is used which includes model previous predictions up to step \( k+i \) and future model inputs \( u'(k+i) \) generated by the controller.

**Predictive control objective functional**

Given a prediction horizon \( H_p \) and a control horizon \( H_c \) with \( H_c < H_p \), the control samples \( u^* \) are determined by minimising the following cost functional

\[
J = \sum_{i=1}^{H_p} \left( r(k+i) - \left( \hat{y}(k+i|k) + e(k) \right) \right)^2 + \beta(k) \sum_{i=0}^{H_c-1} \left( \Delta u(k+i|k) \right)^2
\]

where \( b(k) \) is a relative weight for the incremental input \( \Delta u(k) \) from which the full input is obtained as \( u(k) = u(k-1) + \Delta u(k) \). The model error \( e(k) \) is included in the functional (3) to enhance its robustness. In fact whenever a disturbance acts upon the process output, the feedback signal is equal to that disturbance and is not influenced by the control action, since the FPC predictions are computed with a model not affected by the disturbance. Therefore, the signal is simply subtracted from the set-point. Since the FPC is the fuzzy version of a Internal Model Reference controller (Morari and Zafiriou, 1989), when exact model matching is obtained, additive output disturbance can be cancelled.

**Branch and Bound**

Over the prediction horizon \( H_p \) each control value which is applied to the model at time \( k \) would produce an explosive sequence of future alternatives over the control horizon \( H_c \) (Bellman’s “curse of dimensionality”), as shown in Figure 4.

To limit the alternatives along the decision tree a Branch-and-Bound (B&B) algorithm
(Horowitz and Sahni, 1978) has been used: at each step \(i (i = 1, ..., H_c)\) the input \(u^*(k+i)\) is selected which minimizes the current functional value (up to step \(i\)) \(J^i\) given by

\[
J^i = \sum_{l=1}^i \left( \left( \hat{y}(k + l|k) - r(k + l) \right)^2 + \beta(k) \left( \Delta u(k + l - 1|k) \right)^2 \right)
\]  

(4)

The step-by-step B&B selection procedure is shown in Figure 5. At the \(i\)th step, a generic \(j\)th branch is selected if the cumulative cost so far \(J^i\) plus a lower bound \(J_L\) accounting for the remaining steps up to \(H_p\) is lower than an upper bound of the total cost \(J_U\), i.e. \(J^i + J_L < J_U\). In the present application \(J_U = \infty\) and \(J_L = 0\) are assumed so that for each path the algorithm compares and updates the current cost with the terminal cost \(J(H_p)\). This solution is suitable when a constant or slowly varying set-point is to be tracked.

**Dynamic fuzzy grid**

The Branch and Bound procedure implies that the inputs are selected from a discrete grid of values. As the process is driven towards the reference value, the resolution of the grid should be increased to improve the controller accuracy. A dynamic fuzzy discretization is used to adjust the grid spacing as the search proceeds. New input samples \(u^*\) are selected from the grid domain

\[
u(k + i) = u(k + i - 1) + \omega_j(k) \quad \omega_j(k) \in \Omega_k = \{ \omega_j(k) | j = 1,2,...,N \}
\]

(5)

where \(\omega_j(k) \in \Omega_k\) represents the incremental control action which is selected in the allowable set \(\Omega_k\). This set is adjusted using a scale factor \(\gamma(k) \in [0,1]\) which depends on the activation of simple fuzzy rules, forming a dynamic grid

\[
\Omega_k = \gamma(k) \cdot \left\{ 0, \lambda_l u_k^+, \lambda_l u_k^- | l = 1,2,...N \right\} \quad \lambda_l = \frac{1}{2^{l+1}}, \quad l = 1,2,...N
\]

(6)

where \((u_k^+, u_k^-)\) are the bounds in which the \(u(k)\) sample may vary. The fuzzy adaptive parameter \(\gamma(k) \in [0,1]\) is the output of a fuzzy rule-based algorithm fed by the tracking error and its rate of change according to an empirically tuned empirical rule matrix such as the one to the right.

**Application to carbon dosing control**

The FPC algorithm has been applied to the carbon dosing problem. As a first step, the predictive model was trained with the modified ASM2d model with the dry-weather input benchmark time series. By trial and error it was found that the most suitable model structure was one with three linear consequents each including three past nitrate values \(S_{NO_3}\) and one input \(Q_c\) namely

**Figure 4** Decision tree in the selection of the optimal input over the prediction horizon \(H_p\), illustrating the curse of dimensionality. The dashed arrows indicate the optimal input sequence

**Figure 5** The Branch and Bound decision tree in determining the cost increment from the \(i\)th step to the \((i+1)\)th step
connected with an equal number of fuzzy rules based on the cluster centroids

\[ \begin{align*}
\dot{S}_{NO_3}(k+1) &= 2.09S_{NO_3}(k) - 1.72S_{NO_3}(k-1) + 0.59S_{NO_3}(k-2) - 0.47Q_c(k) + 0.42 \\
\dot{S}_{NO_3}(k+1) &= 0.94S_{NO_3}(k) - 0.23S_{NO_3}(k-1) + 0.23S_{NO_3}(k-2) - 0.24Q_c(k) + 0.35 \\
\dot{S}_{NO_3}(k+1) &= 2.06S_{NO_3}(k) - 1.65S_{NO_3}(k-1) + 0.58S_{NO_3}(k-2) - 0.04Q_c(k) + 0.1 
\end{align*} \]  
(7)

It could be objected that the training regime is rather different from the controlled operation, but it should be remembered that the latter is included in the former and that the controller is continually tuned, so the model obtained at the end of the training phase is just the initial condition for the closed-loop operation which readjusts the model to the new regime. For closed-loop operation, the following FPMC parameters were assumed: \( H_p = 3; \ H = 2; \beta = 0 \) and the following grid adaptation mechanism was adopted

\[ \Omega_k = \gamma(k) \left[ \frac{u_k^-}{16}, \frac{u_k^0}{4}, \frac{u_k^+}{4}, 0, \frac{u_k^-}{16} \right] \]  
(9)

An example of the performance of the predictive fuzzy controller is shown in Figure 7. It can be seen that the dosing flow \( Q_c \) is somewhat higher than in the open loop case (Figure 6), but this saving is obtained at the expense of a much higher nitrate output concentration.

**Set-point selection**

From a practical point of view, it is important to place the fuzzy predictive controller in a wider framework of plant operation management. In this context, the tracking precision is not as important as the resulting level of nitrate being released. Also, the organic carbon consumption is to be taken into account, as it represents an additional operating cost. Hence, the controller should be used to keep the concentration of released nitrate at a level consistent with the required standard and yet limit the amount of added carbon. Now the problem is shifted to determining the most appropriate nitrate set-point value \( S_{NO_3}^{sp} \) which minimises the amount of added carbon and still comply with the required nitrogen release limit. It

![Figure 6](imageurl)  
*Figure 6* Open loop training of the carbon–nitrate fuzzy model. The dotted lines represent the feedforward signals.
should be reminded that, with the given plant configuration, the amount of nitrogen released is not equal to the nitrate set-point $S_{NO_3}^{sp}$ as a consequence of dilution.

To evaluate this effect, the average nitrogen output was computed and plotted against the anoxic tank set-point and total added organic carbon. The results are shown in Figure 8, assuming a nitrogen limit of 10 mg/l, as introduced by the E.C. directive 271/1991 and subsequent Water Framework Directive. This plot should serve as a guideline to select the highest set-point, and hence the lowest carbon consumption, which still comply with the directive limit. It appears that a set-point of about 1.2 mg/l is the most suitable.

**Conclusion**

In biological nitrogen removal processes, organic carbon has to be supplied to the heterotrophic denitrifying bacteria. If not enough COD is available in the input sewage, an external carbon source has to be artificially provided. To minimise this operational cost the dosing should be controlled in order to follow the time-varying process requirement and to supply the minimum amount for which the discharge standards are met. The former goal can be pursued through automatic control of the dosing flow around a nitrate set-point, whereas the latter can be regarded as a higher-level problem of selecting the most suitable set-point from an economical point of view. The solution proposed here is based on a fuzzy predictive controller (FPC). Though predictive control has been in use for many years, and a fuzzy version was proposed by Babuska (1998), the present version has some innovative features in the fuzzy predictive model formulation and the adaptive grid of the Branch and
Bound search. Its advantages are the ease of tuning and its adaptive capabilities, even in the presence of a highly nonlinear dynamics. With respect to other adaptive controllers applied to the same problem and described in a previous paper (Marsili-Libelli and Manzini, 2000) the FPC resulted in a more robust operation, being less sensitive to disturbance and easier to tune.

Regarding the higher-level problem of set-point selection, it should be considered that the main goal is not so much the precision of nitrate control as the minimum added carbon which still keeps the ef fluent in the required limit. This upper set-point value can be found by running the controlled plant simulation with increasing set-points and computing the total effluent nitrogen. The highest set-point for which the standard is met will therefore represent the most economical operational setting. For the case study at hand, it was found that this limit is about 1.2 (mgN-NO₃/l). Of course this value holds for the present case only, but a general procedure has been outlined which can be applied to any process of this kind.

References