FUZZY MODELLING OF THE COMPOSTING PROCESS

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Abstract
Successful composting implies a careful monitoring of the batch process, which includes two phases: active composting and curing. In the first phase, a succession of microbial activities transform the organic material into progressively biodegradable matter, whereas during the second phase, stabilization ensues. The key to efficient composting lies in the definition of an appropriate temperature batch curve and in an effective control system to track such a profile. The control system should rely on a process model in order to predict the batch behaviour, in terms of temperature, and adjust the air supply accordingly.

Keywords. Composting, Fuzzy models, Process control

INTRODUCTION
Composting is a biological treatment process that is largely applied to the biodegradable fraction of solid waste to dispose of rubbish and obtain useful fertilizer. This process is attractive for its low energy consumption, limited technological requirements and commercial value of end-product. The composting process consists of the biochemical degradation of organic materials through a controlled microbial aerobic decomposition process with the formation of stabilized organic materials that may be used as soil conditioners and organic fertilizers.

Composting is the biochemical degradation of organic materials to a safe, humus-like material through a three-phase microbial aerobic process with the formation of stabilized organic materials that may be used in agriculture as an organic fertilizers (Haug, 1980; Negro et al., 1999). The main factors in the control of a composting process include environmental parameters, such as temperature, moisture content, pH and aeration (Liang et al., 2003; Kulcu and Yaldiz, 2004). The main products of biological metabolism are carbon dioxide, water, and heat and aeration has the twofold effect of providing oxygen for the biological reactions and removing the heat which they produce. For this reason temperature is regarded as the single most important parameter being both an indicator of process condition and the main reaction conditioner (Kulcu and Yaldiz, 2004). For this reason temperature profiles during the batch are highly important and the control system is designed to follow a profile which is considered to yield the most effective reaction in terms of disinfection, moisture content and degradation (Mason and Milke, 2005; Mason, 2006). Two typical temperature profiles are shown in Figure 1, where two main phases can be seen: the initial mesophilic phase when disinfection occurs, followed by a thermophilic phase, when the biological reaction in the compost are completed. The former refers to the initial high-temperature period when the biological reactions are very fast a require a considerable amount of oxygen. The latter refers to the degradation of the less reactive substances, with a decreased oxygen requirements. It is important that during the first phase the temperature is high enough to guarantee disinfection. For this reason the
ECOMAS reference curve is higher in the first few days. The purpose of the control system is to follow the reference temperature profile of Figure 1 as closely as possible during the whole batch. For this reason a process model is required, in order to predict the future temperature evolution and modulate the air flow accordingly.

![Typical temperature profile during the composting batch.](image)

**Figure 1.** Typical temperature profile during the composting batch.

**FUZZY MODELING OF THE COMPOSTING PROCESS**

A fuzzy approach is pursued in developing a dynamical model of the composting process and its performance is evaluated with batch data from full-scale composting plants. The reason for preferring a fuzzy model to a deterministic one (see e.g. Sole-Mauri et al., 2007) lies in the inherent complexity of the composting process, with a multi-stage succession of fungi and bacteria colonizing the organic material. This succession is difficult to model in deterministic terms (Sole-Mauri et al., 2007), whereas identifying a fuzzy model from operational data is reasonably simple, though it provides a moderate degree of insight into the process biochemistry. Nevertheless, a preliminary exploratory phase is useful to gain a knowledge of the basic process behaviour in terms of temperature profile.

The approach used in this paper follows the guidelines of Babuska (1998) and Abony (2003) in setting up a Sugeno fuzzy model with clustered antecedents and least-squares estimation of consequents. This approach proves very useful in modelling complex systems, for which rule-based conventional fuzzy models would result in an explosive proliferation of rules. A multi input single output (MISO) model was set-up using the product space fuzzy clustering based on the Gustafson-Kessel algorithm using data from a 400 m$^3$ biotunnel for which airflow/temperature coupled data were available for the whole batch duration, with the former being expressed as the ON/OFF setting measured at the air valve and the output being the temperature measured inside the compost pile.

**Structure of the fuzzy model**

When approximating a complex nonlinear dynamical system, a single linear model is often insufficient, especially when the nonlinear system has several operating regimes with significantly differing characteristics. Examples of this behaviour can be found, for example, in biotechnical processes, where the system behaviour strongly depends on biomass viability. This problem can be solved by the decomposition of the system operating range into a set of regimes to which local linear models are associated. The global model is obtained through a smooth interpolation of the local models, giving the most weight to the one which is thought to best approximate the system in the current operating point and less weight to those which are considered to be far from the present condition. A classifier is needed to decide which linear model is best approximating the current system operation and select the interpolating weights.
accordingly. This classifier should provide a set of smooth interpolating functions to combine
the individual local models. This approach (regime identification and multi-model blending) can
be naturally recast into the fuzzy set theory which provides an easy way for solving both the
regime identification (fuzzy clustering) and the model blending problem (consequent models
identification).

Model development

The Nonlinear ARMAX model (NARMAX) has often been used to approximate nonlinear systems
representing its behaviour by nonlinear mapping of past inputs, outputs, and noise terms to
obtain future outputs (Chen and Billings, 1989). For this application a multi input single output
deterministic model with measurement noise is used to describe the relationship between
airflow and temperature in the biotunnel, namely

\[
y(k) = f \left( y(k-1),...,y(k-n_y),u(k-1),...,u(k-n_u) \right) + e(k)
\]  

(1)

where \( y, u \) and \( e \) represent the process output, input and measurement noise respectively.

The problem of decomposing the system behaviour into a number of significant operational
modes has been considered from the fuzzy point of view by Hathaway and Bezdek (1993) and
then by Babuska (1998) and Abony (2003). In many instances, like in this application, the
features \( z(k) \) describing the operational state may differ from the input/output data, though
they are generally strictly related to them. Let \( z(k) \) represent the current operating point in the
operational space \( Z \) defined as a function of the past input/output samples,

\[
z(k) = f \left( y(k),y(k-1),...,y(k-n_y),u(k),u^T(k),u^T(k-1),...,u^T(k-n_u) \right) \in Z
\]  

(2)

where in general \( n_y \neq n_z \) and \( n_u \neq n_z \). Once a suitable features space \( Z \) has been defined, the
problem of mode recognition consists of determining the most appropriate complete partition
\( Z_i \subset Z \) of all the process modes and attach a local linear model to each of them. This
approximation problem, as posed by eqs. (1-2), can be recast into a fuzzy context in a quite
natural way. From the structural point of view, TS models (Takagi and Sugeno, 1985) can
indeed be viewed as a fuzzy extension of the deterministic interpolation concept. Further, a
fuzzy decomposition yields a higher computational efficiency since the subsets \( Z_i \) are no longer
disjoint, but contribute to the global model through their degrees of matching. Lastly,
partitioning of the operational space \( Z \) into \( Z_i, \ i = 1...M \) can be obtained in a systematic way
through fuzzy clustering (Babuska, 1998; Abony, 2003). In addition to requiring that the
resulting fuzzy memberships must be smooth functions, for each point \( z(k) \in Z \) defined by eq.
(2), to ensure model completeness there must exists at least one cluster \( Z_i, \ i = 1...M \) for
which the corresponding membership function \( \mu_z(z_i) > 0 \). The current operational state of the
system can then be expressed by the set \( \mu_z(z_i) \mid i = 1,...,M \) of the condition memberships.

Though the basic model structure adopted here is based on TS models, the implication-based
antecedent is substituted by a cluster structure to reduce model complexity and maintain the
concept of regime decomposition. Thus the Fuzzy Clustering theory of Bezdek (1981), further
developed by Babuska (1998) and Abony (2003) can be applied. With these premises, let the
process be approximated in the generic i-th regime by a linear discrete-time model

\[
\hat{y}_i(k+1) = a_{y,i}y(k) + ... + a_{y,i}y(k-n_y) + \beta_{a,i}u(k) + ... + \beta_{a,i}u(t-n_u) + \gamma_i
\]  

(3)

Then the overall model response can be obtained by combining the collection of models (3)
through the degree of membership of the clusters defined in the operational space
\( Z: z \subset Z_i \rightarrow \mu_{i,k} \), namely
IF z_i ∈ Z_i THEN \[ \hat{y}_i(k + l) = \alpha_{0,i}y(k) + ... + \alpha_{n_y,i}y(k - n_y) + \beta_{0,i}u(k) + ... + \beta_{n_u,i}u(t - n_u) + \gamma_i \] (4)

where \( n_y \) and \( n_u \) are integers defining the dimensions of the consequent linear models (it is assumed that all models have the same dimensions).

The model output is obtained a fuzzy combination of the individual linear model outputs resulting in a nonlinear interpolation, or defuzzification

\[
\hat{y}(k + l) = \frac{\sum_{j=1}^{M} \hat{y}_j(k + l)\mu_{z_j}(z_k)}{\sum_{j=1}^{M} \mu_{z_j}(z_k)}
\] (5)

The complete fuzzy modelling structure is shown in Figure 2, showing the preliminary feature extraction block, followed by clustering, which produces the required degree of truth to be applied to each consequent model. After defuzzification according to eq. (5) the model prediction is obtained.

**Features extraction**

Assuming that the basic process element is the OFF/ON aeration cycle, there are three basic modes in which the temperature may respond to the aeration, as described in Figure 3. The relevant variables defining each pattern are the temperatures at the beginning (\( T_a \)) and at the end (\( T_b \)) of the OFF portion of the cycle and at the end of the ON cycle (\( T_c \)), the maximum temperature swing (\( T_d \)) and the difference between the initial and final temperatures, i.e. \( \Delta = T_c - T_a \). The three possible responses are shown in Figure 3 and represent the features upon which the input/output data are clustered.

**Figure 3.** The three possible temperature patterns defining the operational models to be clustered.

In addition to the direct observations, suggesting that the modes of Figure 3 are indeed exhaustive, the optimal number of clusters has been searched with the XB criterion (Xie and
Beni, 1991) which considers the ratio of the total within clusters variation and separation of cluster. The optimal number of cluster should minimize the value of the index

$$XB(c) = \frac{\sum_{i,j}^c (\mu_{ij})^2 \|x_i - v_i\|^2}{N \min_i\|x_i - v_i\|^2}$$

(6)

where $c$ is the number of clusters, $v$ are their centroids and $N$ the number of input/output process data. The selection obtained with the XB criterion was confirmed also by other clustering criteria such as the fuzzy hypervolume, the average partition density and the average cluster flatness (Babuska, 1998).

![Figure 4. Four differing partition indicators agree in indicating an optimal partition into three clusters.](image)

**Model building**

Several operational records were clustered using the Gustafson-Kessel fuzzy algorithm (Babuska, 1998) using the features of Figure 3 in order to extract information for the model-building process. Clustering the process data with the Gustafson-Kessel algorithm (Babuska, 1998) provided the following values of the cluster centers (centroids).

<table>
<thead>
<tr>
<th>Features</th>
<th>Centroids</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta T_k = T_p - \min(T_c, T_a)$</td>
<td>$\Delta_k = T_c - T_a$</td>
</tr>
<tr>
<td>$T_{\max}(k)$</td>
<td></td>
</tr>
<tr>
<td>$Z_1$</td>
<td>1.90</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>1.81</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>1.14</td>
</tr>
</tbody>
</table>

![Table 1. Centroids of the three clusters describing the operational modes.](image)

The original temperature and pressure data were split in two separate data streams. The pressure data provided information about the aeration cycle, duration and on/off ratio (duty cycle). The temperature data were first processed with a denoising wavelet decomposition; from its second approximation $A_2$ the cluster features of Figure 3 were extracted according to the following definitions eq. (7), with $k$ being the cycle number, which is assumed as the basic time unit. The model inputs are composed of information on the aeration cycle (duration and duty cycle) and features extracted from the process ($\delta T_k$ and $\Delta_k$). These data are fed into the model identifier which, given the model order, calibrates the consequent sub-models, as in eq. (4) minimizing the sum of squared difference between the observed and computed $T_{\max}(k)$ at each cycle.

![Figure 5. Cluster selection along the normal composting batch.](image)
Figure 6. Structure of the model-building process to infer the maximum temperature reached in each cycle.

The complete model is shown in eq. (8) and its calibrated parameters are reported in Table 1. The actual model output is obtained from the three model outputs via defuzzification, according to eq. (4)

\[
\text{if } z_k \in Z_1 \text{ then } \hat{y}_1(k+1) = \alpha_{1,1}y(k) + \alpha_{2,1}y(k-1) + \beta_{1,1}u_1(k-2) + \beta_{2,1}u_2(k-3) + \\
+ \beta_{3,1}u_3(k-4) + \beta_{4,1}u_4(k-2) + \beta_{5,1}u_5(k-3) + \beta_{6,1}u_6(k-4) + \\
+ \beta_{7,1}u_7(k-2) + \beta_{8,1}u_8(k-2) + \beta_{9,1}u_9(k-2) + \gamma_{1,1} \\
\text{if } z_k \in Z_2 \text{ then } \hat{y}_2(k+1) = \alpha_{1,2}y(k) + \alpha_{2,2}y(k-1) + \beta_{1,2}u_1(k-2) + \beta_{2,2}u_2(k-3) + \\
+ \beta_{3,2}u_3(k-4) + \beta_{4,2}u_4(k-2) + \beta_{5,2}u_5(k-3) + \beta_{6,2}u_6(k-4) + \\
+ \beta_{7,2}u_7(k-2) + \beta_{8,2}u_8(k-2) + \beta_{9,2}u_9(k-2) + \gamma_{1,2} \\
\text{if } z_k \in Z_3 \text{ then } \hat{y}_3(k+1) = \alpha_{1,3}y(k) + \alpha_{2,3}y(k-1) + \beta_{1,3}u_1(k-2) + \beta_{2,3}u_2(k-3) + \\
+ \beta_{3,3}u_3(k-4) + \beta_{4,3}u_4(k-2) + \beta_{5,3}u_5(k-3) + \beta_{6,3}u_6(k-4) + \\
+ \beta_{7,3}u_7(k-2) + \beta_{8,3}u_8(k-2) + \beta_{9,3}u_9(k-2) + \gamma_{1,3} 
\]

The model output is obtained by defuzzification of the three model outputs, according to eq. (4)

\[
\hat{T}_{\text{max}}(k+1) = \frac{\sum_{i=1}^{3} \hat{y}_i(k+1) \mu_{Z_i}(z_k)}{\sum_{i=1}^{3} \mu_{Z_i}(z_k)} 
\]

Table 2. Model parameters after calibration.

<table>
<thead>
<tr>
<th>regimes</th>
<th>a_1</th>
<th>a_2</th>
<th>b_1</th>
<th>b_2</th>
<th>b_3</th>
<th>b_4</th>
<th>b_5</th>
<th>b_6</th>
<th>b_7</th>
<th>b_8</th>
<th>b_9</th>
<th>b_10</th>
<th>γ_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z_1</td>
<td>-0.032</td>
<td>0.892</td>
<td>-0.145</td>
<td>-0.174</td>
<td>1.321</td>
<td>-2.511</td>
<td>2.505</td>
<td>-0.964</td>
<td>-6.856</td>
<td>6.720</td>
<td>-4.003</td>
<td>5.203</td>
<td>-0.913</td>
</tr>
<tr>
<td>Z_2</td>
<td>1.224</td>
<td>-0.248</td>
<td>0.335</td>
<td>-0.126</td>
<td>-0.141</td>
<td>3.656</td>
<td>-1.731</td>
<td>0.848</td>
<td>7.125</td>
<td>-6.987</td>
<td>6.352</td>
<td>-5.004</td>
<td>-8.889</td>
</tr>
<tr>
<td>Z_3</td>
<td>0.792</td>
<td>0.676</td>
<td>-3.715</td>
<td>2.047</td>
<td>-1.372</td>
<td>19.232</td>
<td>7.120</td>
<td>-5.944</td>
<td>-0.134</td>
<td>-0.010</td>
<td>-2.416</td>
<td>-0.202</td>
<td>2.733</td>
</tr>
</tbody>
</table>
The model identification was performed using the algorithm just outlined and the results shown in Figure 7 were obtained. Figure 8 shows the agreement between the experimental data and the calibrated model response.

**Figure 7.** Fuzzy model identification results. The succession of prevailing memberships is consistent with the process evolution during the batch.

**Figure 8.** Agreement between the experimental temperature data and the identified model output.
CONCLUSIONS

This paper has presented a dynamical model of the composting process based on a hybrid fuzzy model with clustered antecedents and linear deterministic consequents. The model is based on air flow input data and temperature output data drawn from operating composting cells. The resulting data clusters are in agreement with the main process phases, including active composting and curing. The reason behind the model is that efficient composting requires accurate tracking of an appropriate temperature curve for which a model-based control system is required to adjust the air supply accordingly. The model inputs are the duration and duty cycle of the aeration cycle and features extracted from the process. With these data the model identifier determines the consequent sub-models by minimizing the sum of squared difference between the observed and computed maximum temperature at each cycle. The resulting model is in good agreement with the observed behaviour and is the first building block towards the required automatic control system.

REFERENCES